Wavy composite is a new form of constrained layer damping that uses opposing sinusoidal waveforms in composite constraining layers to greatly enhance damping. Damping measurements as high as 26%, with the stiffness of aluminum and titanium are shown.

This research is focused on evaluating a test system capable of determining the stiffness and damping of structural materials with high inherent damping. Time-temperature superposition can be applied to create a master curve from 1 Hz to 10,000 Hz that completely describes material performance as a function of frequency and temperature. This paper reveals how the accuracy of the low frequency portion of the nomograph is affected by very small errors in the test data that translate to large errors in the Arrhenius frequency shift constants. Detailed sensitivity analyses confirm the accuracy of test results, especially the magnitude of the damping and stiffness results. However, this paper shows how the frequency at which a given value of damping and stiffness occur is subject to increasing error when extrapolated to lower frequencies. Arrhenius time-temperature superposition constants are determined experimentally. Error associated with extrapolating to very low frequencies is discussed.


1. INTRODUCTION

The purpose of this paper is to present some recent developments in testing wavy composite. The basic axial test procedure was described in Testing Wavy Composite published in the Spring 2001 SAMPE conference proceedings (Pratt, et al. 2001). In this paper, in depth analyses of the Arrhenius method of creating material nomographs is presented with recommendations for obtaining greater accuracy at low frequencies.
Characterizing the dynamic performance of damped composite materials presents a number of challenges. First, wavy composites exhibit frequency and temperature dependence in their stiffness and damping properties. Therefore, the material properties must be described as a function of frequency and temperature. Second, no test method exists capable of testing over a range of frequency broad enough to capture the entire material master curve, so it is difficult to characterize the performance of the composite with a single test apparatus. Third, most dynamic test methods don’t allow testing of the material properties directly. Typically, a test material is combined with one or more other materials in some “structure” which is then tested. Pure material properties can be difficult to extract from test results for the structure (Sperling 1989).

An axial test apparatus developed by Patterned Fiber Composites, Inc. overcomes most if not all of these challenges. Individual tests characterize the material performance from 1000 Hz to 2000 Hz. The test specimens are tested over this narrow frequency range but at a number of different temperatures. The time-temperature superposition principle is used to generate a master curve of the material performance at a single reference temperature over a very broad range of frequencies (approximately 1 Hz to $10^4$ Hz or greater). Because the specimens are tested axially, the test algorithm determines axial stiffness and damping independent of the area moment of inertia, cross section shape, or other structural properties. Wavy composite can be rolled onto a mandrel without cutting fibers along the length of the tube, so edge and cut fiber effects associated with flat samples are eliminated (Pratt, et al. 2001).

**2. UNKNOWNS AND LIMITATIONS**

Application of time temperature superposition requires one major assumption. It must be assumed that there is some interdependence of frequency and temperature in the material. Temperature is, in essence, a measure of the kinetic energy of atoms, which is proportional to their velocity. When vibrations are excited in a structure, the frequency of oscillation is also essentially a measure of the velocity of the particles in the material, so in a broad sense frequency and temperature will always be interconnected (Sperling 1989). Obviously, the properties of materials such as metals and ceramics do not change appreciably in the range of temperatures and frequencies in which we commonly use them. The same is true for composites, which typically exhibit properties that are essentially constant over their useful frequency and temperature range.

However, the properties of many polymers, such as viscoelastic, change dramatically with relatively small changes in temperature and/or frequency. Wavy composites are then a combination of a viscoelastic that is strongly frequency and temperature dependent, and a carbon fiber composite that is not. It will be assumed that the overall composite structure will take on the properties of the polymer and exhibit material properties that depend strongly on frequency and temperature. The smoothness of the resulting stiffness and damping curves after applying time-temperature superposition confirms the validity of this assumption. In fact, test data presented herein will show that the properties of wavy composite are indeed dependent on frequency and temperature, with a frequency-temperature properties very similar to that of the viscoelastic material.

The test method was presented in a previous SAMPE paper and will not be discussed further in this paper. However, for reference, the actual experimental apparatus can be seen in Figure 1.
Figure 1 shows a photograph of the axial test stand. The tube is well isolated from spurious vibrations, since it is only attached through the actuator. Since the mass of the end fixtures, and the mass and the dimensions of the tube can be accurately measured, the only unknowns are the stiffness and damping of the tube material. These can be determined from the transfer function of the two accelerometers with very good accuracy (Pratt, et al. 2001, Pratt and Allen 2001).

3. SENSITIVITY ANALYSIS

The primary purpose of this paper is to discuss the accuracy of the test method used to determine the axial material properties of wavy composite sample tubes, to determine the sources of error, and assess their impact on the accuracy of material master nomographs. Analysis was performed to determine sensitivity to measurement error of physical properties, errors in test data, and errors attributable to other sources.

First a sensitivity analysis was performed to determine the effect of inaccuracies in measuring tube properties on the accuracy of the test algorithm. The analysis program requires that the tube cross sectional area, mass, and length be input, as well as the mass of the end piece attached to the free end. To evaluate sensitivity, one set of data was re-analyzed a number of times while one of the properties mentioned above were increased or decreased by five percent. The percent change in stiffness and damping were then determined.

The effect was essentially one to one for cross sectional area and tube length. Five percent error in either parameter results in five percent error in stiffness. Damping was unaffected. A five percent increase in cross sectional area results in a 5% decrease in stiffness, where a 5% increase in length results in a 5% increase in stiffness. These error effects were linear.

There is some uncertainty as to whether the actual length of the tube should be used, or if the length between the center of mass of the end pieces should be used. In the results presented here, the actual length has always been used. If some effective length actually results in a more accurate stiffness measurement, then based on this sensitivity analysis, the stiffness results presented here would be off by 2-3% for the worst case.
For illustration purposes, the effect of a 5% error in tube mass and end mass will be studied to assess the impact of these possible errors, though errors of this magnitude are highly improbable.

Figure 2 shows the resulting stiffness and damping curves when the tube mass is increased or decreased by 5%. A simulated transfer function was constructed for a tube of similar dimensions and weight with a constant 15% damping and a constant stiffness of 60 GPa. This resulted in a resonant peak at 1610 Hz. The results were then nondimensionalized by dividing the actual frequency by the resonant frequency. Figure 2 shows that the error caused by 5% error in the tube mass can become quite large at frequencies well removed from the resonant frequency, however, over most of the 0.2*f_r to 2*f_r range, the error is attenuated. The error in the damping prediction begins to increase after the resonant frequency and reaches 5% (loss factor of .1575) at about 2*f_r. Over most of the test range the stiffness and damping are less sensitive to errors in tube mass, though it is important to remember that error in the tube mass may change the slope of the stiffness and damping versus frequency curves.
Figure 3: Stiffness and damping with ±5% error in End Mass

Figure 3 shows the stiffness and damping results when the end mass is varied. The trend is similar to that for tube mass, but the error increases at frequencies above the resonance instead of decreasing. Five percent error in end mass resulted in only 1.4% error in stiffness. Studies have shown that the ratio between the masses is really the governing factor (Pratt, et al. 2001).

Figure 4: Converged stiffness and damping with ±5% error in magnitude.
An analysis was also performed to determine the sensitivity of the complex stiffness to errors in the transfer function (magnitude and phase). Figure 4 shows the results of the analysis when the magnitude of the transfer function is artificially increased or decreased by 5%. Error in the magnitude and phase may be due to spurious vibrations in the test stand or sensor noise. In reality, it is very unlikely that the error in the transfer function would be this high over the whole range of frequencies, though this analysis will serve to illustrate the effect of error in the transfer function.

The horizontal scale has been normalized by dividing the frequency by the resonant frequency ($f_r$). Note that the +5% and –5% curves are not symmetrical. Positive error in the magnitude has a more severe effect than negative error at low frequencies. Figure 4 suggests that the most reliable data will be found from $0.8f_r$ to $1.6f_r$. The effect of error in the phase angle will be examined next.

Figure 5: Stiffness and damping results with 5% change in phase angle.

Figure 5 shows the result when 4.58 is added to or subtracted from the phase angle. Since the phase angle starts at 0°, multiplying the phase by 1.05 has a greater effect at higher frequencies where the phase angle approaches 180°. A more fair approach is to add 4.58, 5% of the phase at the middle value, though this represents a larger error percentage at frequencies well above or below the resonance. The effect of the phase angle on damping is very dramatic, though the data is almost unaffected at the resonance. Once again, phase errors as large as those shown above are very unlikely, though Figure 5 is useful to help in understanding performance of the test stand. It would appear that the stiffness data would be more reliable than the damping data, since the damping data is seen to diverge wildly with errors in the phase angle. However, the spurious vibrations that appear as noise in the test data usually consist of a magnitude disturbance that covers a large frequency range and a phase disturbance that appears as a narrow spike, which usually only affects the data over a small range of frequency. For this reason the phase and thus damping results may actually be more accurate over a large frequency range (Pratt, et al. 2001).
The results are concluded as follows. The numbers in parenthesis represent the approximate error seen in testing due to each specific source.

Damping is subject to the following errors:

- **At Resonance:**
  - Damping is completely unaffected by errors in any of the tube properties. (0%)
  - Damping is mostly unaffected by errors in the transfer function. (<1%)

- **Away from Resonance:**
  - Damping is completely unaffected by errors in cross sectional area or length.
  - Errors in tube mass and end mass are attenuated, but become significant above two times the resonant frequency.
  - Errors in the phase angle have a significant effect.

The stiffness measurement is subject to the following errors:

- **At Resonance:**
  - Errors in cross sectional area and length have a one to one effect. (5%)
  - Errors in the end mass or tube mass are attenuated. (<1%)
  - The stiffness is sensitive to errors in the transfer function.

- **Away from Resonance:**
  - Errors in cross sectional area and length have a one to one effect.
  - Errors in the end mass or tube mass become more significant, but are still attenuated below two times the resonant frequency.
  - Error in the stiffness due to error in the transfer function increases away from resonance.

It is evident that the axial test stand is very accurate at resonance. Most of the possible sources of error have no effect on the data at resonance. Errors in the tube mass, end mass, or transfer function can result in small errors in the data away from resonance, though the data at resonance is very reliable.

4. **TIME-TEMPERATURE SUPERPOSITION**

The principle of polymer frequency-temperature superposition related to the determination of wavy composite nomographs was presented previously (Pratt, et al. 2001). This section will focus on the effects of the errors discussed in section 3 on the master nomograph for a particular wavy composite material combination.

As discussed previously, the wavy composite-viscoelastic combination tends to take on the time-temperature superposition characteristics of the viscoelastic alone, and when available the shift constants of the viscoelastic alone can be used to give a good approximation. This section will explore this in more detail. First, a tube made with 7.5 cm (3.0 in.) 308 wavy material using 0.28 mm (0.011 in.) thick Avery-Dennison FT1191 viscoelastic will be analyzed (named TR16).
The linear fits are exceptional for this tube above 278°C, as seen in Figure 6 (colder curves in Figure 6 are shown in the relative order of the legend with colder curves on the bottom of the chart). Below 278°C the data begins to appear noisy. The slope of the curve fit lines seems representative of the data at every temperature, though at 18.9°C the data is very noisy and it is difficult to say what the true slope should be.

Figure 7 shows the linear fit used to determine the Arrhenius shift constant, $q$. Once again, the data seems to diverge slightly at lower temperatures (upper end of the curve). In this case, the
low temperature divergence is probably due to noise in the data at low temperatures, as seen in Figure 6. The curve fit results in $q = 8357$.

![Modulus & Damping Nomogram for TR16](image)

Figure 8: TR16 Data shifted according to Arrhenius and Avery-Dennison constants

Figure 8 shows the stiffness and damping data shifted using both Arrhenius and Avery-Dennison’s constants. The Arrhenius constant places the damping peak at about 20 Hz. The Avery-Dennison shift constant place the damping peak at about 85 Hz. Which one is correct? The Arrhenius curve visually appears smoother than the Avery-Dennison curve so one would conclude that the tested curve fit is more accurate. In terms of temperature, the spacing between these damping peaks is about 10°C and much of the discrepancy seen above could be due to inaccuracies in the temperature measurement, or variations in the temperature of the tube from top to bottom.
One more tube will now be examined. This is a 5.0 cm (2.0 in.) 308 tube made with 0.28 mm (0.011 in.) thick Avery-Dennison FT1191 viscoelastic (named TR17). First, the curve fits through each segment of damping data will be analyzed.

As shown Figure 9, the curve fits through this data are all excellent, even for the data at low temperatures. Visual inspection of the data and linear fit gives no reason to suspect that these curve fits might give erroneous results. Figure 10 shows the linear fit which yielded $q = 8971$. The line fits the test data almost perfectly.
Figure 11 shows the data for TR17 shifted using both the tested data and the manufacturer supplied data. Once again, the tested Arrhenius constants place the damping peak at a significantly lower frequency than the Avery-Dennison constants. The resulting master curve looks significantly smoother when the tested Arrhenius constants are used. The horizontal spacing between the two damping peaks represents a temperature shift of 6-78°C.

In this section, the challenge involved in extrapolating to low frequencies using time temperature superposition has been demonstrated. When the Arrhenius shift constants are determined experimentally, the resultant master curve tends to be shifted to lower frequencies than expected.

The differences in the shift constants seen previously may be explained by a number of factors. First, the Avery-Dennison viscoelastic curve fits were quite noisy, and there may be error in the viscoelastic shift constant reported. Second, the test stand allows for small temperature gradients between the top and the bottom of the tube as heat is conducted away through the piezoelectric actuator, into the steel base. The temperature measured at the center of the tube may not always be representative of the average temperature of the tube. Finally, error in the measured transfer...
function can affect the slope of the damping and stiffness curves, and thus affect the accuracy of the shift constants obtained from the data, especially if there is a bias in the test set-up as discussed in Section 4.3.5.

Finally, the extension of the master curve to lower frequencies is subject to significant error because the shift required for high temperature, low frequency data is a cumulative of all previous shifts. If error is biased due to some anomaly in the test set-up, the result would have a tendency to cause large variations in the shift constants as will be seen in the next section.

The data presented in this section suggests that the actual Arrhenius shift constant for the composite-viscoelastic combinations examined here is 8000-9000. This large variation seems excessive at first, though differences of \( \pm 500 \) in the shift constant produce a shift in the damping peak that is almost indiscernible.

If it is assumed that wave angle, period, and thickness do not affect the shift constant, comparisons between the various test samples would be unaffected by the shift constant used. The agreement between the test data and the FEA analysis using the same shift constant suggests that this assumption is valid (Pratt, et al. 2001, Pratt and Allen 2001, Pratt, et al. 2001). The analysis in the next section will illustrate the sensitivity of the shift constant to very small errors in the measured transfer function.

Simulated test data was generated using a curve fit of the results for a 7.5 cm (3.0 in.) 308 wavy composite tube. The simulated test data was generated by curve fitting the complex stiffness and damping as a function of frequency and temperature. This data was then used to construct a transfer function from 500 to 3200 Hz at discrete temperatures. 1% error was then added or subtracted from the magnitude and phase data to simulate noise in the data. This data was then analyzed using the same algorithms used in testing, resulting in stiffness and damping as a function of frequency at a number of temperatures. Time temperature superposition was then used to find the best-fit Arrhenius shift constants for this data.

Figure 12: Curve fits on segments of simulated data with 1% error
Figure 12 shows the linear fits used on each segment of simulated damping data. Notice that there is no visible sign of error. A 1% error has been imposed on the transfer function over the entire test frequency range, so the error only appears in the slope of the damping curve.

Figure 13: Arrhenius Fit on Simulated Data with 1% Error.

Figure 13 shows the linear fit used to determine the Arrhenius constant for the simulated data with +1% error imposed on the transfer function. Once again, there is nothing in the linear fit that would indicate that the error is there. With +1% error the fit yields $q = 3816$. With −1% error the fit yields $q = 8352$. The Arrhenius constant used to generate the data was $q = 5555.56$.

Figure 14: Resulting Shifted Curves with +/- 1% Error in Transfer Function

Figure 14 shows the shifted stiffness and damping curves obtained using the Arrhenius shift constants for the erroneous data. Imposing only 1% error on the transfer function has resulted in large shifts in the master curves. This suggests that small amounts of noise in the transfer function may be causing the discrepancies seen previously in Figure 11.
The data presented in this section reveals an interesting fact. The stiffness and damping value is unaffected by time-temperature superposition, and has been shown to be very accurate, especially at resonance. However, the frequency at which a particular value of stiffness and damping exists is in question because of errors in the slope of the data that determines the frequency shift constant. Mechanical tests accurately determine the stiffness and damping of the tube and can be performed between 1000 and 2000 Hz for any reasonable temperature.

Three points are required to define a second order curve exactly. Since the damping curve is parabolic in the useable range of interest, three points where the damping and frequency value are known would be necessary to complete an accurate nomograph. A room temperature test is unshifted by the Arrhenius equation, so the far right portion of the curve can be trusted. The peak value of damping can be accurately determined from the curve and represents a known second point. However it is the frequency of the damping at room temperature and at a very low frequency that is unknown because of the limitations in testing. Using the procedures outlined in this chapter, this low frequency point in the curve can be determined using shifted data but it has been shown that the shifted data at low frequencies is very susceptible to error. If a single low frequency test was available to “anchor” the lower end of the curve, the shift constant could be adjusted to agree so that every point in between would also be known with high accuracy.

Future testing is recommended at some very low frequency (ie 1 to 10 Hz). This may even be possible on existing tensile equipment. This could serve to anchor the damping master curve at low frequency. Another alternative involves determining time-temperature superposition constants experimentally on a large body of experimental data until the uncertainty in the shift constant is reduced to an acceptable level. Once again, it is important to remember that despite the difficulty in determining the time-temperature superposition shift constant, the value of the stiffness and damping at low frequency is not in question, only the exact frequency at which that stiffness and damping occurs.

5. CONCLUSIONS

The axial test stand is an effective tool in characterizing the performance of stiff, highly damped composites. Investigations into all of the possible sources of error have shown that the test stand is accurate and robust. Sensitivity analyses have shown that the data at resonance is highly reliable, though the data is somewhat sensitive to error away from resonance. The magnitude of this error away from resonance is normally very small.

Time-temperature superposition has been applied to create master stiffness and damping curves over a very broad frequency range. The Arrhenius time-temperature superposition constant has been shown to be highly sensitive to error in the slope of the stiffness and damping data. Experimentally determined shift constants vary between 5000 and 9000, while the value for the viscoelastic alone is reported as 5555.56. The scatter seen in testing has been shown to be significantly less than occurs when a 1% error is imposed on the magnitude and phase curves. Because of this high sensitivity to error, the low frequency properties determined through time-temperature superposition are susceptible to error. It is important to remember that despite the difficulty in determining the time-temperature superposition shift constant, the value of the stiffness and damping at low frequency is not in question, only the exact frequency at which that stiffness and damping occurs.
Wavy composite takes on the properties of the viscoelastic, and the viscoelastic curve fit constants appear to be a reliable approximation that can be used to shift test data and form master nomograms. Further testing at very low frequencies or on a larger body of data will determine the time-temperature superposition shift constant more accurately.

Methods for testing in bending and in torsion have revealed that the material behaves similarly in bending and torsion, though further research will be necessary to make these methods as robust and reliable as the axial test.

6. REFERENCES


