A Method for Generating Pseudo Single-Point FRFs from Continuous Scan Laser Vibrometer Measurements

Matthew S. Allen[1] and Michael W. Sracic

Department of Engineering Physics
University of Wisconsin-Madison
535 Engineering Research Building
1500 Engineering Drive
Madison, WI 53706
[1]Email: msallen@engr.wisc.edu

Abstract:

The laser Doppler vibrometer (LDV) has been recognized as the instrument of choice when non-contact vibration measurements are required, and is also capable of acquiring measurements with high spatial detail; one only need redirect the laser beam via computer controlled mirrors to acquire data for additional measurement points. However, since most LDV systems are only capable of acquiring data for one point at a time, testing times tend to be quite long. This problem is formidable for structures with low natural frequencies, such as aircraft, space structures or civil structures, which would require prohibitively long testing times if LDV were used. One typically must use an automated excitation device, such as an electromagnetic shaker, to excite the structure while hundreds of time histories are acquired, but these excitation devices can modify the structure, which is precisely what one was trying to avoid by selecting a non-contact measurement device. One also encounters difficulty if the structure under test changes with time or the desired input is not reproducible. This paper presents a method by which one can sweep the laser continuously over the structure as it vibrates and decompose the measured velocity into a collection of point frequency response functions, each related to the response at a point on the structure traversed by the laser. The measurements can be processed using standard modal analysis software to identify the mode shapes of the structure from a single transient response. The method makes no assumptions regarding the shape or properties of the surface and only requires that the laser scan periodically and that the structure be vibrating freely, such as due to excitation by an instrumented hammer. The method is demonstrated experimentally on a free-free beam, identifying the first nine mode shapes of the beam at hundreds of points from a few free response measurements. This represents a two-order of magnitude reduction in the time needed to acquire measurements with the LDV.

1. Introduction

After the Laser Doppler Vibrometer (LDV) was invented a number of years ago, its remarkable utility was quickly recognized, especially when the structure under test would be modified by attaching traditional contact transducers. Laser vibrometers have been employed to measure vibration on rotating components, such as spinning hard disk drives and rotating machinery [1]. State of the art LDV systems include computer controlled mirrors (scanning LDV) so one can control the position of the laser spot on the structure automatically, and align the measurement points with images of the structure. These allow one to set up measurements quickly and to automatically locate measurement points with high spatial accuracy. Other recent advances include measurement of 3D motion and the ability to acquire the spatial position of measurement points in three dimensions.

One significant limitation of current LDV systems is that they require an exceedingly long time to acquire test data when the structure of interest has modes with low natural frequencies. Time is expensive in the realm of modal testing. The continuous scanning technique presented in this paper could greatly reduce the cost of acquiring measurements because the responses at hundreds of pseudo-points are obtained simultaneously. This is especially important for expensive aircraft and satellite systems, for which test personnel and prototype hardware may cost hundreds of thousands of dollars per day. For example, Brillhart *et al*. [2] recently reported on the ground vibration test of a small aircraft that required two crews working around the clock for 10 days to acquire data at 415 points on the structure. While such a data set is large compared to what was reasonable twenty years ago, it may be vastly inadequate to validate a finite element model containing hundreds of target modes and modeled with millions of degrees of freedom. One can imagine the cost that would be required to increase the size of the test data base substantially using conventional methods.

Modal Testing theory dictates that, whether transient or broadband excitation is used, the time required to acquire broadband measurements at a point on a structure is dictated by the length of the structure's impulse response, which is typically many cycles of the lowest frequency mode of vibration. For example, if a structure's most slowly decaying mode has a natural frequency of 2.0 Hz, with 1% damping, the corresponding decay constant is 0.02 s^{-1}, and the time, T, required for the response of this mode to reduce to 1% of its initial amplitude is T = -ln(0.01)/0.02 = 230 sec. Hence, at least 3.8 minutes of the structure's response must be acquired at each measurement point. When such a record length is multiplied by the number of averages performed and by hundreds of response points, the test time can quickly become prohibitive using standard laser vibrometry [3]. Furthermore, many important transient events are difficult or costly to reproduce exactly, for example, a blast input or slip in a complex mechanical joint. A traditional laser vibrometer could capture the response at only one point for such a response, so it is abandoned usually for such an application in favor of accelerometers or other transducers that are cheaper to use in parallel.

In recent years, Stanbridge, Ewins and Martarelli [4] have sought to develop a new mode of operation of the laser vibrometer that has the potential to address this issue. They propose scanning the laser spot continuously over the structure while measuring, and have derived methods by which the operating deflection shape of the structure can be derived from the measured signal. The method, known as Continuous Scan Laser Doppler Vibrometry (CSLDV), was actually first presented by Sriram, Hanagud and Craig [5]. Recent works by Stanbridge, Ewins and Martarelli suggest that CSLDV could be used to identify the three-dimensional velocity vector for a vibrating surface (conical scan), the displacement and rotations of a point, and semi-continuous operating deflection shapes over a line or a surface [4, 6, 7]. These are promising steps forward toward solutions to high impact problems in structural dynamics, yet some significant limitations remain.

Stanbridge, Ewins and Martarelli [4] presume that the structure is excited such that it executes pure sinusoidal motion, and then derive the operating deflection shape at a single frequency from a CSLDV measurement. Two methods have been described, one which derives the operating shape over the laser's trace path using demodulation techniques [8], another which derives a polynomial series representation of the mode shape by relating the polynomial coefficients to the amplitude of the response at various frequencies (combinations of the scan frequency and the vibration frequency) [4]. Unfortunately, these methods have yet to fully prove their utility. When a structure's response is sinusoidal as the methods require, one can simply step the laser from point to point and use fast signal processing methods in combination with the known drive frequency to find the amplitude and phase of the response at that frequency (i.e. Polytec®'s "fast scan" feature). It remains to be seen whether Stanbridge, Ewins and Martarelli's CSLDV approach will prove any faster than such a discrete approach [9]. In any event, whether the continuous scan or "fast scan" approach is employed, the process must typically be repeated at a multitude of excitation frequencies before modal analysis can be performed to fully characterize the structure. If the structure has close natural frequencies, then multiple input locations may also need to be used, so the net decrease in test time may be minimal. Furthermore, the requirement that the structure undergo sinusoidal motion necessitates the use of a shaker or other contacting excitation device, and hence the experiments are more complex to set up and require more expensive hardware. If the structure of interest is light or delicate, there may also be significant risk of modifying or damaging the structure when attaching the excitation device.

Sriram and his associates explored a different approach. They utilized broadband random excitation while sampling synchronous with the laser scan speed to generate conventional FRFs from the CSLDV response [5, 10]. Unfortunately, the signal processing they proposed appears to have neglected the effect of the input between each period of the laser's path, which may have contributed to the low coherence that they observed. One could perhaps adapt the signal processing methods they proposed to account for the between sample input if one can assure that the input spectrum is negligible above the scan frequency, yet it appears that they did not do this. Their efforts in this area seem to have ceased a decade ago.

Stanbridge, Martarelli and Ewins also investigated another flavor of CSLDV in which the free response of a structure was measured while sweeping the position of the laser spot periodically [11, 12]. They used relatively slow scan speeds and a zoom-FFT to get a detailed view of the spectrum near a natural frequency, and observed that the coefficients in their polynomial representation for each mode shape [6] could be determined from the amplitudes of peaks in the spectrum of the CSLDV signal. Figure 1 shows the spectrum that they published. The natural frequency of the beam is near the peak labeled '0'. Each of these peaks was then fit to the standard representation for a single mode, and the coefficients were collected and processed yielding what appears to be a plausible 3rd mode shape for a cantilever beam [12].

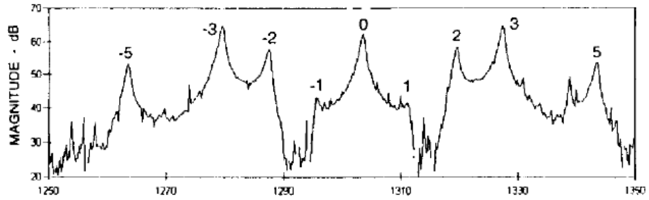

Figure 1: Zoom FFT spectrum near the 3rd natural frequency of a cantilever beam using CSLDV – From Stanbridge, Martarelli & Ewins [12]

In principle, this method could be used to extract both the mode shapes and natural frequencies of a structure all from a single continuous-scan measurement of its free response. This could dramatically reduce the amount of time required to perform a modal survey, allowing one to obtain detailed, non-contact measurements from low frequency structures in a very short time. Unfortunately, the approach in [12] falls short of this ideal, because considerable effort must be expended to apply a zoom FFT for each mode. Also, one must fit a modal model to each peak in the response spectrum, and there could be many of these per mode with varying signal strengths.

This work presents a new mode of continuous scan laser vibrometry for transient response data that addresses some of these issues. Methods that were recently developed to identify the parameters of Linear Time-Periodic (LTP) systems are applied to free response data measured using a continuously scanning LDV, decomposing the response into pseudo-FRFs at the points traversed by the laser. The collection of pseudo-FRFs can be processed using standard modal parameter identification routines, identifying the mode shapes and natural frequencies of the system. Because the pseudo-FRFs are similar to standard FRFs for a linear time-invariant system, the processing is readily accessible to anyone who is familiar with conventional modal analysis. One can also average measurements and add data from additional input locations in much the same way one does for time invariant systems. Furthermore, by casting the problem in the framework of a linear time-periodic system, one can more easily develop methods to account for the between scan-period inputs that were apparently neglected in the work of Sriram, and extend the method to forced vibration. The methods presented here could greatly reduce the time and expense required to collect spatially rich measurements from low frequency structures. This could allow one to obtain detailed models of low frequency systems, models which cannot be obtained within realistic time and budget constraints using conventional methods.

This paper proceeds as follows. Section 2 presents the theory underlying the proposed analysis method. The method is applied to experimental data from a simple beam suspended by bungee cords in Section 3. Section 4 presents some conclusions.

2. Theory

Allen and Ginsberg [13] recently presented a pair of methods that can be used to identify models of linear time-periodic (LTP) systems from free response measurements. The motivation for the research was initially to detect cracks in rotating machine elements. The anisotropy introduced by a crack in a rotating part can lead to parametric resonance and failure of the machine, and this phenomenon cannot be predicted using a linear-time invariant system model. The identification methods were derived based on Floquet theory [14, 15] and discrete system theory [16].

We begin with the signal $y(t)$ measured by a scanning LDV on a freely vibrating, linear, time-invariant structure, which can be expressed as a sum of decaying exponentials in terms of the modal parameters of the structure as follows

$$y(t)=\sum_{r=1}^{2N}\phi(x)_r\,C_r e^{\lambda_r t}$$
$$\lambda_r=-\zeta_r\omega_r+i\omega_r\sqrt{1-\zeta_r^{\,2}}\ ,\ \lambda_{r+N}=-\zeta_r\omega_r-i\omega_r\sqrt{1-\zeta_r^{\,2}} \qquad (1)$$

where λ_r is the rth complex eigenvalue, which is expressed in terms of the rth natural frequency ω_r and damping ratio ζ_r. The corresponding mode vector $\phi(x)_r$ depends on the position x of the laser beam along its path. This path could involve motion in three dimensions in a general case. C_r is the complex amplitude of mode r, which depends upon the structure's initial conditions, or on the impulse used to excite the structure if such an excitation is employed.

The laser is assumed to traverse a known, periodic path of period T_A, such that $x(t) = x(t+T_A)$, or with The scan frequency $\omega_A = 2\pi/T_A$, resulting in the following representation.

$$y(t) = \sum_{r=1}^{2N} \phi(t)_r \, C_r e^{\lambda_r t}$$
$$\phi(x(t))_r = \phi(t)_r = \phi(t+T_A)_r \tag{2}$$

This is identical to the expression for the free response of a Linear Time Periodic (LTP) system given in [13].

The following sections discuss how to process these responses in order to determine $\phi(t)_r$ and λ_r. Two methods are presented. The preferred method for data processing, dubbed the Multiple Discrete Time Systems (MDTS) method in [13, 17] is described in the following section. The second method, dubbed the Fourier Series Expansion (FSE) method, was found to be similar to the approach employed by Stanbridge, Martarelli and Ewins, as will be discussed in Section 2.2.

2.1. Multiple Discrete Time Systems (MDTS) Method

One can eliminate the time dependence of the mode shapes in eq. (2) by sampling the response only at instants in which $x(t)$, is equal to some constant $x(t_i) = x_i$ (and hence $\phi(t)$ is also constant). This is the essence of the Multiple Discrete Time Systems (MDTS) method, which recognized that the response thus sampled is identical to the response of a linear time-invariant system. The method is most useful when one samples faster than the scan speed, so one can measure the response at numerous points by simply setting the data acquisition* system to acquire samples at a sample increment of $\Delta t = T_A/N_A$ where N_A is an integer specifying how many samples are acquired per period of the periodic system. Current LDV systems are capable of sampling at rates of up to a few MHz, thousands of times faster than realistic scan rates $1/T_A$. One can then define sampled responses.

$$\begin{aligned} y_0 &= [t_0, t_0 + T_A, t_0 + 2T_A, \cdots, t_0 + N_c T_A] \\ y_1 &= [t_1, t_1 + T_A, t_1 + 2T_A, \cdots, t_1 + N_c T_A] \\ &\cdots \\ y_{P-1} &= [t_{P-1}, t_{P-1} + T_A, t_{P-1} + 2T_A, \cdots, t_{P-1} + N_c T_A] \end{aligned} \tag{3}$$

Each of these is identical to the free response that one would measure at each position x_i. For example, for a starting time of t_1, where $0 < t_1 < T_A$, the pseudo-LTI response $y_1(t)$ at time instants $t = t_1 + kT_A$ where $k = 0,1,\ldots$ is given by

$$y_1(t_1 + kT_A) = \sum_{r=1}^{2N} \phi(t_1)_r \, C_r e^{\lambda_r (t_1 + kT_A)} \ . \tag{4}$$

These can be transferred to the frequency domain using the Discrete Fourier Transform (DFT or FFT), resulting in a set of pseudo-Frequency Response Functions (FRFs). The term pseudo is used because the residues in eq. (4) (and in its Fourier transform) differ from the residues of a true FRF by a scale factor. In other respects these pseudo-FRFs can be treated exactly as is done for traditional FRFs. A collection of these responses for various initial times spanning $(0,T_A]$ share the same eigenvalues, differing only in their modal amplitudes, so they can be processed using a global parameter identification routine to find the system's eigenvalues and mode shapes (up to a scale factor C_r). The resulting mode shapes encompass all of the points that the laser has swept during the time response.

When using the MDTS method, one can incorporate additional measurements to minimize errors and address close natural frequencies, analogous to what is done for linear time invariant systems. For example, if the force that initiated the free vibration is repeated at the same location, one would expect the constants C_r for all

* One could sample at a different rate and then interpolate to obtain the response at the desired instants. A variety of methods have been developed for doing this and are implemented in commercial hardware.

of the modes to be roughly the same although all possibly differing by a common factor, so the measurements could be averaged to reduce measurement noise. Input at a different point would likely change the proportionate contribution of the modes and hence their constants C_r, so the set of responses can be treated as an additional column of the pseudo-FRF matrix and processed appropriately to identify modes with close natural frequencies [18-20].

One will notice that the responses in this collection are not sampled synchronously, so one must account for the time delay between each spatial sample to obtain accurate mode shapes, as discussed in the appendix of [13] and in [17].

It is important to recognize that the MDTS procedure has the effect of aliasing any natural frequencies above $\omega_{max} = \omega_A/2$. Modes with natural frequencies higher than ω_{max} appear at lower frequencies in the MDTS responses. The natural frequencies identified while processing the MDTS responses must be corrected to obtain the true natural frequencies. This could be done using a point measurement (or a small number of point measurements) to supplement the CSLDV measurements. Another approach is simply to avoid exciting modes above ω_{max}. Many of the modes identified in this work were aliased, yet a simple method was derived for finding their true natural frequencies. As discussed in the appendix of [13], the true (unaliased) natural frequency of each mode is needed to account for the inter-sample delay when finding its mode shape. The authors have found empirically that the mode shapes obtained in this process tend to be highly complex if the natural frequency used in the process is not correctly unaliased. A simple routine was created that adds various integer multiples of ω_{max} to the identified eigenvalues and searches for that which gives in the greatest modal phase co-linearity. For lightly damped structures such as that used here, this has proven to be a very robust way of determining the true (unaliased) natural frequencies.

2.2. Fourier Series Expansion Method (FSE)

One can gain additional insight into the CSLDV process by examining the spectrum of the measured signal in eq. (2). Because the scan pattern $x(t)$ is periodic, the mode shapes can be expanded into a Fourier Series as follows,

$$\phi_r(t) = \sum_{m=-N_B}^{N_B} B_{r,m} \exp(im\omega_A t) \tag{5}$$

where it is assumed that only the coefficients from $-N_B$ to $+N_B$ are significant. Substituting into eq. (2) and moving the summations to the outside results in the following.

$$y(t) = \sum_{r=1}^{2N} \sum_{m=-N_B}^{N_B} B_{r,m} \exp\left(\left(\lambda_r + im\omega_A\right)(t - t_0)\right) \tag{6}$$

This is mathematically equivalent to the impulse response of an LTI system with $2N(2N_B + 1)$ eigenvalues

$$\lambda_r + im\omega_A. \tag{7}$$

The amplitude of the response of each of these apparent modes is determined by the magnitude of $B_{r,m}$, the mth Fourier coefficient of the rth mode shape. Hence, the bandwidth of a CSLDV signal is governed by both the natural frequencies $|\lambda_r|$ of the excited modes and the number of terms required to represent their mode shapes in a Fourier series.

Stanbridge and his associates utilized a similar representation to identify the modes of a cantilever beam from transient CSLDV responses [12]. They employed modal analysis to identify each of the coefficients $B_{r,m}$ and then used the coefficients to approximate a power series model for the mode shape. One difficulty associated with their approach is that there may be many, many peaks in the CSLDV spectrum (see Figures 5 and 6). This necessitates either a labor intensive process, or identification algorithms with high order matrix polynomials [21] and the associated computational cost and ill conditioning. Furthermore, the post processing required to determine which peaks correspond to each mode may be daunting. The MDTS method presented in Section 2.1 is easier to implement since the resulting system can be modeled with low order matrix polynomials with many spatial points. A large number of spatial points are easily accommodated using the modal parameter identification methods in [20, 22, 23], for example. Allen has processed thousands of frequency response functions simultaneously using the algorithm in [20].

2.3. Noise and resolution limitations of CSLDV

In principle, one could obtain data from millions of pseudo-points over the surface of a structure from a single sweep of the laser spot if one could sample quickly enough. In practice, the actual resolution that can be obtained will be limited by measurement noise. The dominant noise source in CSLDV measurements will most likely be laser speckle noise. Its effect and characteristics must be considered in order to design a successful CSLDV system. Unfortunately, its impact on laser vibrometer measurements has only recently begun to be appreciated, and efforts at modeling it are only now beginning to appear [1, 24, 25].

Laser speckles appear due to constructive and destructive interference between light scattered from different regions on a surface. They have a complex effect on vibrometer measurements. The speckle pattern is caused by micro-scale surface irregularity, so the pattern translates with the surface, randomly modulating the intensity of the light collected by the photodetector as speckles either appear onto or fall off of the sensing area of the photodetector. The speckle pattern has a significant effect on the phase of the intensity of the laser light and hence on the measured velocity. Rothberg presented an effort at modeling this in [24].

Fortunately, the frequency characteristics of speckle noise can be exploited to minimize its effect on CSLDV measurements. Speckle noise in CSLDV measurements occurs predominantly at the scan frequency and its harmonics (up to the 100th harmonic or higher). The laser's path is periodic, so the speckle noise, which depends on the surface roughness over that periodic path, is also periodic. Martarelli and Ewins suggest a way of spacing the CSLDV scan frequency relative to the vibration frequency to minimize the possibility that the frequencies of interest overlap with speckle noise frequencies [7]; they suggest that such an approach is very important. The MDTS analysis method presented here places all of the speckle noise at the zero frequency line of the spectrum, so it should be easy to distinguish from the meaningful spectrum. That line, which is typically not trustworthy anyway due to offset errors in the data acquisition, is easily discarded or ignored when using the MDTS method.

3. Experimental Application

The proposed continuous scan vibrometry method was evaluated by applying it to a simple, aluminum (Al 6061 T6) beam with dimensions 97.16 by 2.54 by 0.635 centimeters. The beam is light and flexible, and hence it might be modified by attaching contact transducers or excitation devices. The beam was suspended by 2.38mm diameter bungee cords during the test to simulate a free-free condition. The support system was designed so that the rigid body modes would have natural frequencies about ten times lower than the first elastic mode [26], so that the first natural frequency should not shift more than about 1%. Elastic supports were also employed in the x-direction to minimize x-directional swinging and x,y-plane rotation, because these rigid body motions would alter the position of the scan path on the beam and possibly cause the laser spot to fall off the edge of the beam. A photograph and a schematic of the configuration are shown in Figures 2 and 3.

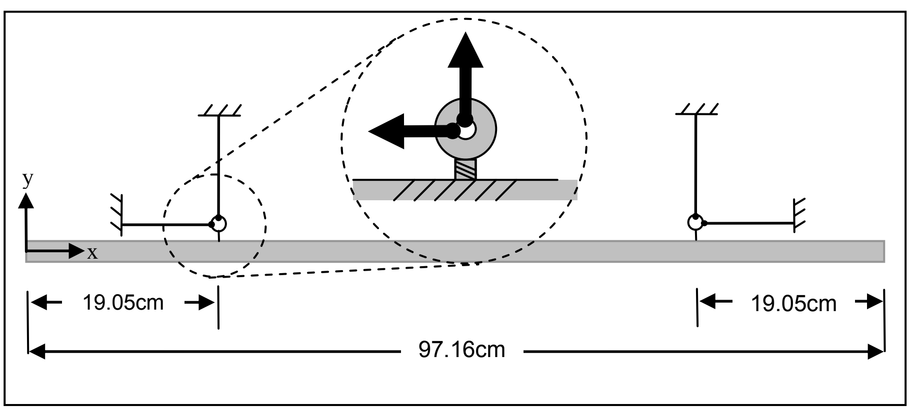

Figure 2: Schematic and dimensions of test setup.

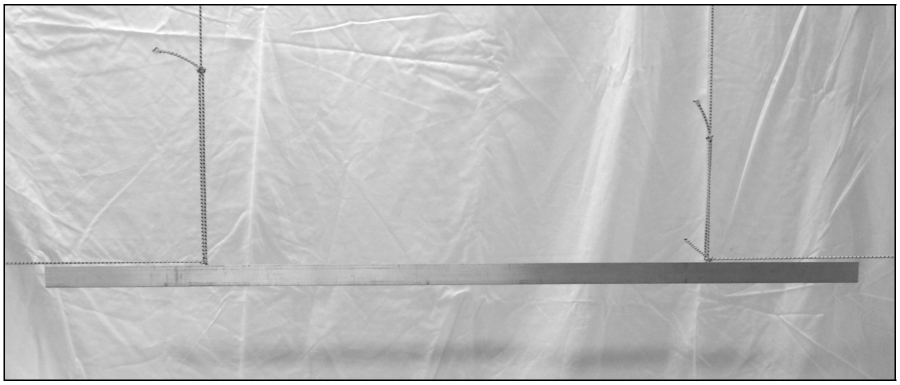

Figure 3: Aluminum beam and support system.

3.1. Hardware Setup

A schematic of the measurement hardware for the experiment is shown in Figure 4, which includes the following:

i) Polytec® PSV-400 80kHz scanning laser vibrometer
ii) Custom cable to externally control the scan-mirror servo motors
iii) Tektronix AFG 3022 Dual Channel Arbitrary Function Generator
iv) PCB Piezotronics modally tuned ICP impact hammer 086C01

The software laser position control was replaced by a user defined voltage signal by replacing one of the vibrometer control cables with a custom cable. A Tektronix AFG 3022 Dual Channel Arbitrary Function Generator was used to generate a 100 Hz sinusoidal signal that was used to drive the x-direction mirror. The y-direction scan-capability was not utilized in this study.

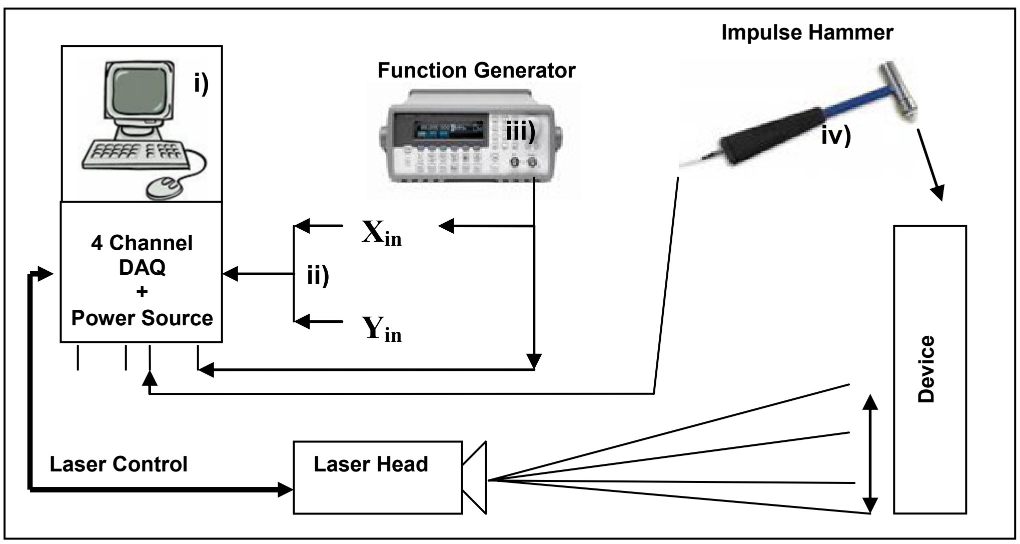

Figure 4: Schematic of Data Acquisition Hardware

When utilizing standard, discrete-point scanning, one selects measurement points graphically on a video image acquired by the laser scan head and the software calculates the mirror voltages required to direct the laser to the desired points. Because this calibration information was not readily available, the amplitude of the signal driving the x-mirror was simply adjusted until the laser traversed all but a few millimeters of the length of the beam. This signal was also fed into the data acquisition system and was later used to determine the location of the laser spot as a function of time. The input voltage v was assumed to be related to the position of the laser spot x by a linear dynamic system such that $X(f_{scan}) = G(f_{scan})V(f_{scan})$, where f_{scan} is the scan speed. The magnitude and phase of $G(f_{scan})$ were determined as follows. The distance between the end of the laser path and the ends of the beam was recorded and used to determine the scale factor relating voltage and laser spot position. Initial tests revealed that there was also a significant phase delay between the voltage sinusoid and the x-position. This was clearly visible in the mode shapes that were found using the MDTS algorithm, because the mode shapes differed for the forward and backward sweeps of the laser. The phase delay was found by adjusting the phase of x relative to v until the mode shapes for the forward and backward sweeps were identical. The gain and phase were thus found to be 15.7mV/cm and -43° respectively at a 100 Hz scan speed.

The LDV vibration signal, mirror drive signal and hammer force signal were recorded for five different impact locations. No attempt was made at averaging to reduce noise in the LDV signal, although this would certainly be beneficial and will be implemented in future studies. Data acquisition was triggered by the force hammer, and then the portions of each of the time records occurring before and during the force pulse were deleted to obtain the free-response of the beam. Sampling rates of 20 KHz and 80 KHz were utilized, although the former was adequate to capture the full bandwidth of the CSLDV signal, and will be used in all of the following. Thirty seconds of response data was acquired, which was the time required for the vibration signal to decay to a small level. An exponential window with a decay constant of 0.25s^{-1} was then applied to the vibrometer signal to reduce the effects of leakage and noise. This was approximately equal to the minimum decay constant of any of the elastic modes, which was found to be optimum in [20] when a free vibration signal is contaminated with Gaussian noise.

3.2. Results

Figure 5 shows the low frequency portion of the spectrum of the signal acquired by the LDV before MDTS processing. Dozens of harmonics are clearly visible, extending out to at least 1500 Hz. The noise floor begins to dominate the measurements above that point. The bandwidth of the part of this signal that protrudes above the noise appears to be less than 2kHz, so the sample rate should be more than adequate. A plastic tip was used to excite the structure; the input spectrum was found to be more than an order of magnitude down at 2 or 3kHz.

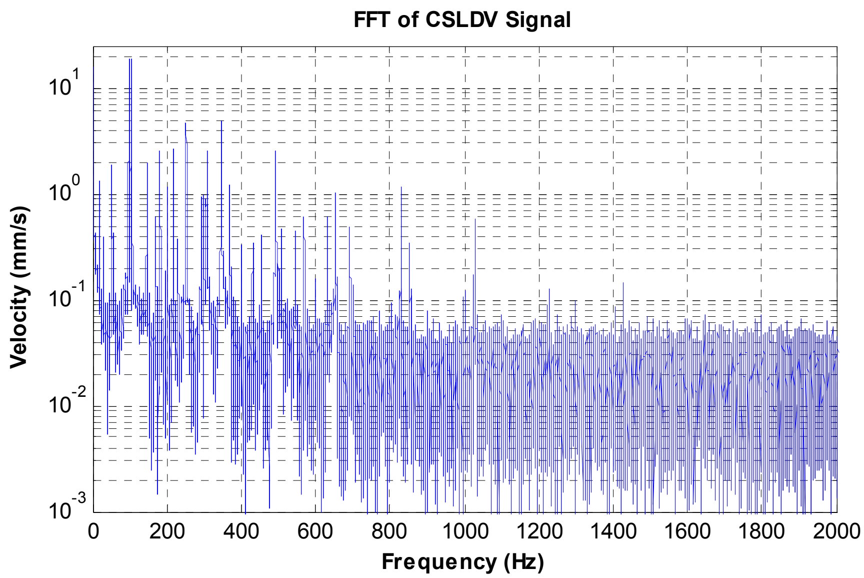

Figure 5: FFT of CSLDV signal for one input

Equation (6) explains that each mode is represented in the CSLDV response as a group of harmonics, spaced by the scan frequency. Figure 6 shows the portion of the spectrum from 0 to 400 Hz. One can readily identify sets of harmonics that are spaced by the scan frequency. For example, there are peaks at 51 and 251 Hz, as well as a very small peak at 151 Hz. Note that the conjugate of each mode (eigenvalue with negative imaginary part) can also be represented in the positive part of the spectrum if positive multiples of f_{scan} are added to it. Hence, the peaks at 149 and 349 Hz can also be attributed to the same mode. It would be quite difficult to process this data using the spectrum in Figure 5, as there are scores of peaks and the noise level is considerable. Sharp peaks are also evident at the scan frequency and its harmonics, the frequencies where speckle noise is expected to be most severe [7].

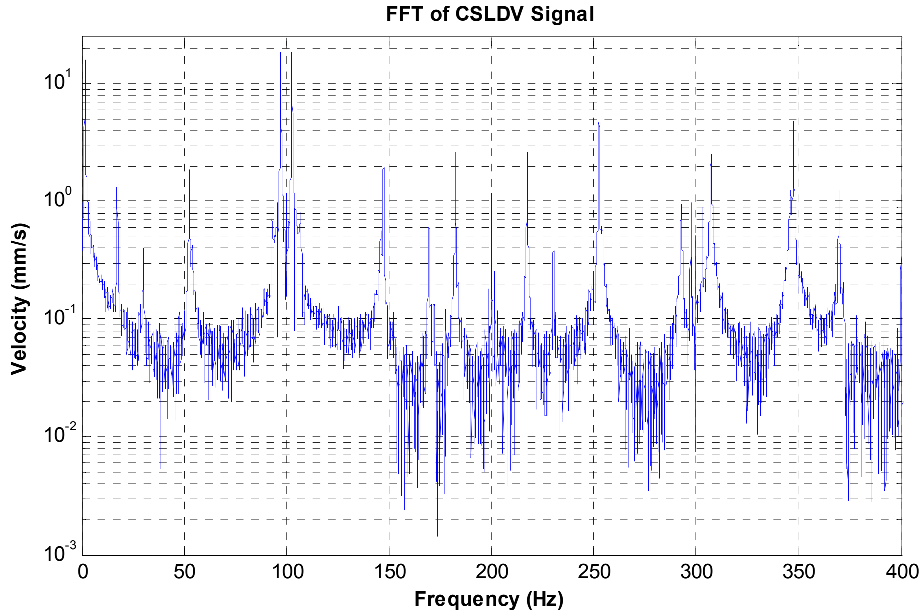

Figure 6: Expanded view of CSLDV signal in Figure 5.

The clock driving the function generator was found to be somewhat out of sync with that of the data acquisition system. To accommodate for this, a sinusoid was fit to the mirror drive signal to determine its frequency, which was found to be consistently 6e-4 Hz higher than its nominal value of 100 Hz. All of the signals were resampled to be synchronous with that frequency using a two step process. First, FFT based interpolation was used to reduce the sample increment by a factor of eight, then linear interpolation was used to find the signal at the desired sample instants. The signal was then decomposed into the components relating to each spatial point using eq. (3), and then transferred to the frequency domain via FFT. The result is a pseudo-FRF matrix comprised of 205 response points and 5 inputs at 1264 frequencies. Figure 7 shows a complex mode indicator function (CMIF) [27] of the MDTS response matrix. Ten peaks are visible in the 50 Hz bandwidth of the signal. The second singular value (green) does not seem to indicate any repeated natural frequencies. Comparing Figures 6 and 7, one can visualize how the peaks in the full spectrum that were mentioned previously fold onto one another to produce a single peak for each mode. This folding is a sort of averaging process, so each peak in the MDTS spectrum (Figure 7) is significantly cleaner than those in the full spectrum (Figure 6). One would expect to be able to obtain a more accurate estimate of the natural frequency of each mode using the peak data in Figure 7 rather than one of the peaks in Figure 6.

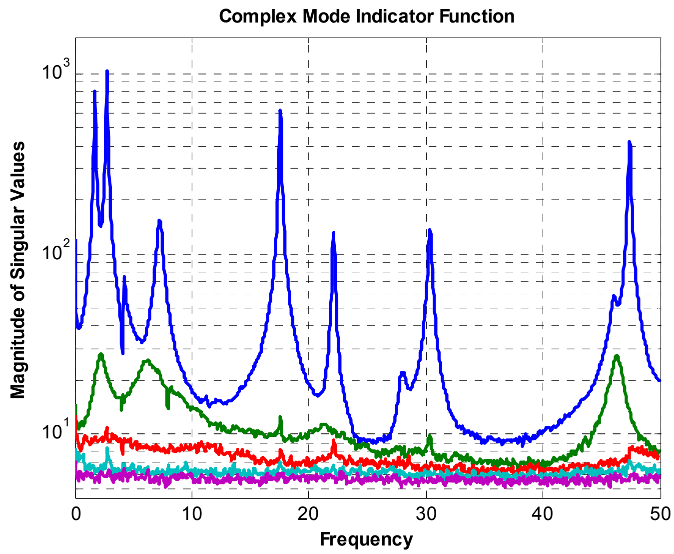

Figure 7: Complex Mode Indicator Function (CMIF) of CSLDV data processed by MDTS method

The matrix of pseudo-FRFs generated from the CSLDV signal was processed using the Algorithm of Mode Isolation [22, 28-31] to identify the natural frequencies and residue matrices (mode shapes) of the modes that were excited. Figure 8 displays a composite (or average magnitude [22]) FRF of the MDTS responses, a composite of the FRF reconstructed using the modal parameters identified by AMI, and a composite of the difference between the two. The data appears to be quite clean and a number of modes well defined. This is partially because the composite FRF averages out noise in the individual FRFs. The individual FRFs were single measurements, not averaged measurements, so they were quite noisy. The modal model identified by AMI agrees very well with the data, as is evidenced by the fact that the composite of the difference between the data and AMI's reconstruction has essentially been reduced to noise. Ten modes were identified by AMI.

Table 1 shows the natural frequencies and damping ratios identified by AMI (after correcting for the exponential window). The natural frequencies are aliased, as discussed in Section 2.1, due to MDTS processing. The unaliased frequencies were estimated using an automated algorithm to search for the value of m in $\lambda_{\text{unaliased}} = \lambda_{AMI} + im\omega_A$ that resulted in the greatest modal phase co-linearity when $\lambda_{\text{unaliased}}$ was used to estimate the mode shapes from the AMI identified residues. The 7th bending mode (27.9 Hz aliased frequency) was weakly excited and its mode shape quite noisy, so manual intervention was required to obtain its unaliased frequency. The analytical frequencies for an Euler-Bernoulli free-free beam of the same dimensions as that used in the test and with nominal material properties are also shown.

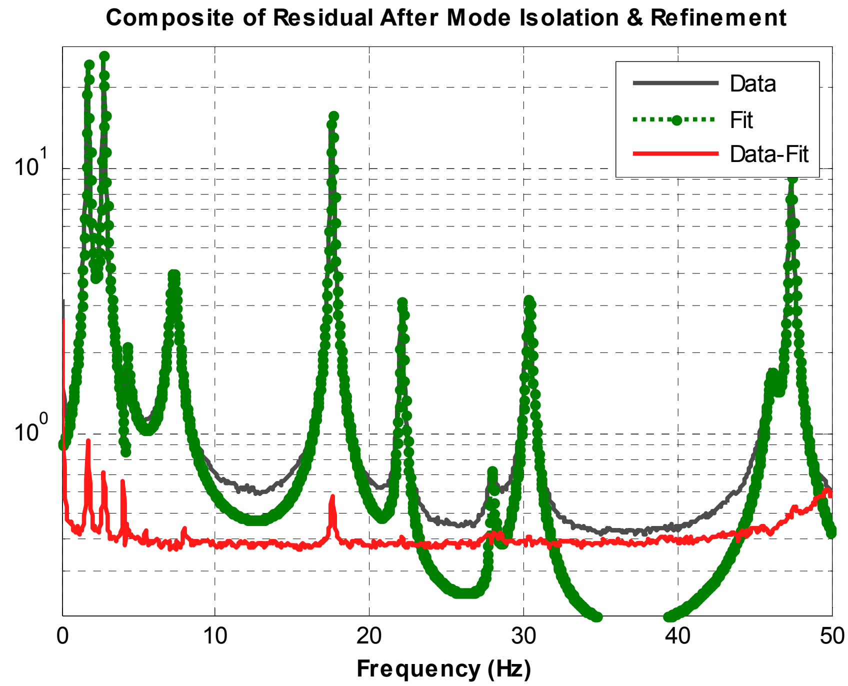

Figure 8: (gray) Composite FRF of the CSLDV data after MDTS processing, (green dotted) Reconstructed Composite FRF after identification by AMI, (red) Composite of the difference between the data and the reconstruction.

Mode #	**AMI Freq (Hz)**	**Damping (%)**	**Unalias Factor** *m*	**Unaliased Freq (Hz)**	**Shape**	**Analytical Freq (Hz)**
1	1.71	1.0	0	1.71	Rigid Body Translation	-
2	2.74	0.51	0	2.74	RB Major axis Rocking	-
3	4.18	0.55	0	4.18	RB Minor axis Rocking	-
4	7.25	2.9	-1	92.75	3rd Bend.	95.1
5	17.6	0.19	0	17.6	1st Bend.	17.6
6	22.1	0.25	3	322.1	6th Bend	328.1
7	27.9	0.25	4	427.9	7th Bend.	436.8
8	30.3	0.38	2	230.3	5th Bend.	234.9
9	46.1	0.57	-2	153.9	4th Bend.	157.3
10	47.4	0.12	0	47.4	2nd Bend	48.5

Table 1: Natural Frequencies of modes identified by AMI.

The mode shapes identified by AMI are shown in Figure 9. As discussed previously, the MDTS process identifies the mode shapes as a function of time. The scan path of the laser was a sinusoid whose amplitude and phase were determined as discussed in Section 3.1. Hence, time can be eliminated and the mode shapes were plotted as a function of position. Because the laser's path is sinusoidal and one-dimensional, the laser spot traces the entire beam twice, so two estimates of the mode shape are obtained, each comprised of over 100 spatial points. The two estimates of the mode shapes are seen to agree closely for all of the identified modes, especially the rigid body modes and lower order bending modes. This provides added assurance that the mode shape estimates are correct, and could also be used to average out noise. Recall that these mode shapes were obtained from only five time histories corresponding to excitation at five different locations on the beam. No averaging was performed when acquiring each of the five measurements, although averaging certainly would have reduced the noise in the measurements. The mode shape of the 7th bending mode was also obtained, yet it was quite noisy, due to this mode being very weakly excited, so it is not shown. If this mode were desired, a harder tip could have been used to bring its response above the noise floor, or an input location could have been sought that maximized its response. The third rigid body mode is also not shown because it involves in-plane rocking, so it looks the same as the 1st mode in this one-dimensional scan. Visual inspection suggests that the identified mode shapes are nearly identical to the analytical shapes for an Euler-Bernoulli cantilever beam, which appear in any standard vibrations text [32]. All of the anti-symmetric modes have nodes exactly at the middle of the beam, suggesting that they are consistent. Also, the nodes nearest the edges of the beam move out with increasing mode number as expected.

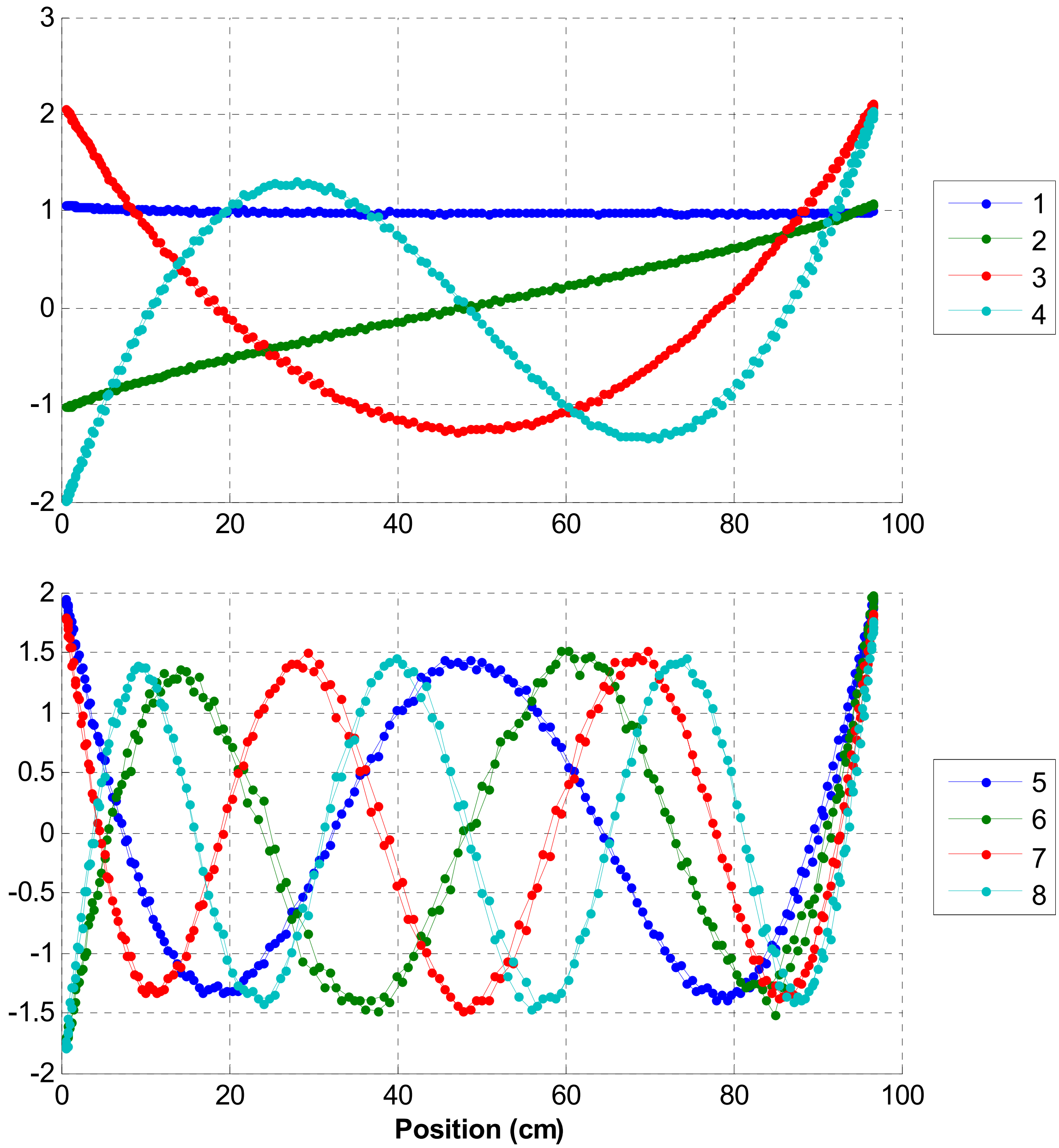

Figure 9: Mode shapes obtained from aluminum beam using CSLDV technique and MDTS processing.

4. Conclusions

Theory was presented whereby the response of a structure measured by a continuously-scanning Laser Doppler Vibrometer (CSLDV) can be processed to identify the natural frequencies and mode shapes of the structure. Two methods were presented. The preferred method, dubbed the Multiple Discrete-Time Systems (MDTS) method, decomposes the CSLDV signal into pseudo-frequency response functions (FRFs) at a multitude of points traversed by the laser spot as it sweeps the structure. These pseudo-FRFs can then be processed using standard modal identification routines, yielding the natural frequencies and mode shapes of the structure (after some post-processing). The MDTS method allows one to incorporate data from multiple input locations so that MIMO modal analysis can be performed to extract modes with close natural frequencies or to assure that all modes are excited. The procedure was demonstrated on a free-free beam, and the resulting mode shapes were found to be visually similar to the analytical mode shapes for an Euler-Bernoulli beam. A second method, dubbed the Fourier Series Expansion (FSE) method, was also discussed, its primary utility being to understand the bandwidth of the CSLDV signal and to troubleshoot any difficulties that might arise.

The proposed CSLDV approach allows for a significant reduction in the time required to test a low-frequency structure. For the example presented here, each of the five time records was 30 seconds long, so approximately 2.5 minutes of vibration data were used to derive the mode shapes shown in Figure 9. This data was decomposed resulting in the mode shape at 205 points, although many pairs of points were nearly replicates of one another, so the actual resolution was approximately 102 points, an average of one point every 0.94 cm. If discrete scanning were used instead, 255 minutes of time data would have been required to assemble a comparable 102 response point, five-input set of measurements. (Both of these time estimates assume that no averaging was performed.) This analysis suggests that the CSLDV approach would reduce measurement time by a factor of one hundred. One should note however, that with the CSLDV approach, many more spatial points could have been acquired in the same amount of time by simply increasing the sample rate, resulting in a ratio of one thousand or more. The actual resolution of the measurements is not set by the number of spatial points, but by the noise floor in the measurements and the resolution required to represent the mode shapes accurately. The mode shapes presented in Figure 9 seem to do this, providing sufficient, yet not excessive, point density for the higher modes of the beam; there are just enough points to allow one to interpolate to nearly any point on the beam. However, if the response at specific locations is critical, one would like to have many more points because they can provide assurance that the model at the critical points is accurate, and not an anomaly due to noise.

There are a number of other practical considerations to consider when comparing this CSLDV method with the discrete-scanning approach. When using the discrete approach, considerable effort would have been required to reposition the shaker for each input location, or else one would have to attach and align multiple shakers. The effort required to do so could be considerable, perhaps the most significant part of the measurement set up. If only one shaker were used, it would be very difficult to repeat the measurements if errors were encountered because of the time and effort required to acquire the measurements. Also, the shaker or set of shakers may have modified the structure causing systematic errors in the mode shapes. The results presented here utilized an impact hammer for excitation, and would not be adversely affected by slight errors in the impact point or direction because the response at every spatial point was acquired for each impact. Of course, there are cases in which it is thought that the time-persistent input provided by a shaker more accurately represents nonlinearity, so that consideration may merit the extra effort required to perform a shaker test.

The proposed methods are best suited for situations in which the transient response of the structure is of interest. They allow one to acquire the response at multiple points simultaneously, which may facilitate studies of structures that can change with time. The methods might also be extended to provide information about transient events such as the response caused by a blast input that is difficult to reproduce, or the response during slip or impact in mechanical joints.

This work considered a one-dimensional structure. Future works will explore optimal scan patterns in two dimensions, as would be required for plate-like structures.

5. Acknowledgements

Sandia National Laboratories provided some of the financial support for this work, under the supervision of Hartono Sumali in the Applied Mechanics Development group. Their support is gratefully acknowledged.

References

[1] S. J. Rothberg, "Laser vibrometry. Pseudo-vibrations," *Journal of Sound and Vibration,* vol. 135, pp. 516-522, 1989.

[2] R. D. Brillhart, T. Deiters, K. Napolitano, A. Giacomin, and C. A. R. Moreira, "GROUND VIBRATION TEST OF AN ADVANCED COMMUTER AIRCRAFT," in *21st International Modal Analysis Conference (IMAC XXI)* Kissimmee, Florida, 2003.

[3] S. Vanlanduit and P. Guillaume, "An automatic scanning algorithm for high spatial resolution laser vibrometer measurements," *Mechanical Systems and Signal Processing,* vol. 18, pp. 79-88, 2004.
[4] A. B. Stanbridge, M. Martarelli, and D. J. Ewins, "Measuring area vibration mode shapes with a continuous-scan LDV," *Measurement,* vol. 35, pp. 181-9, 2004.
[5] P. Sriram, J. I. Craig, and S. Hanagud, "Scanning laser Doppler vibrometer for modal testing," *International Journal of Analytical and Experimental Modal Analysis,* vol. 5, pp. 155-167, 1990.
[6] A. B. Stanbridge and D. J. Ewins, "Modal testing using a scanning laser Doppler vibrometer," *Mechanical Systems and Signal Processing,* vol. 13, pp. 255-70, 1999.
[7] M. Martarelli and D. J. Ewins, "Continuous scanning laser Doppler vibrometry and speckle noise occurrence," *Mechanical Systems and Signal Processing,* vol. 20, pp. 2277-89, 2006.
[8] C. W. Schwingshackl, A. B. Stanbridge, C. Zang, and D. J. Ewins, "Full-Field Vibration Measurement of Cylindrical Structures using a Continuous Scanning LDV Technique," in *25th International Modal Analysis Confernce (IMAC XXV)*, Orlando, Florida, 2007.
[9] M. Martarelli, "Exploiting the Laser Scanning Facility for Vibration Measurements," in *Imperial College of Science, Technology & Medicine*. vol. Ph.D. London: Imperial College, 2001.
[10] P. Sriram, S. Hanagud, and J. I. Craig, "Mode shape measurement using a scanning laser doppler vibrometer," *International Journal of Analytical and Experimental Modal Analysis,* vol. 7, pp. 169-178, 1992.
[11] A. B. Stanbridge, M. Martarelli, and D. J. Ewins, "Scanning laser Doppler vibrometer applied to impact modal testing," *Shock and Vibration Digest,* vol. 32, p. 35, 2000.
[12] A. B. Stanbridge, M. Martarelli, and D. J. Ewins, "Scanning laser Doppler vibrometer applied to impact modal testing," Kissimmee, FL, USA, 1999, pp. 986-991.
[13] M. Allen and J. H. Ginsberg, "Floquet Modal Analysis to Detect Cracks in a Rotating Shaft on Anisotropic Supports," in *24th International Modal Analysis Conference (IMAC XXIV)*, St. Louis, MO, 2006.
[14] P. Montagnier, R. J. Spiteri, and J. Angeles, "The Control of Linear Time-Periodic Systems Using Floquet-Lyapunov Theory," *International Journal of Control,* vol. 77, pp. 472-490, 2004.
[15] G. Floquet, "Sur les Equations Lineaires a Coefficients Periodiques," *Ann. Sci. Ecole Norm. Sup.,* vol. 12, pp. 47-88, 1883.
[16] K. Ogata, *Discrete-time control systems*, 2nd Edition ed. Upper Saddle River, New Jersey: Prentice Hall, 1994.
[17] M. S. Allen, "Floquet Experimental Modal Analysis for System Identificaiton of Linear Time-Periodic Systems," in *ASME 2007 International Design Engineering Technical Conference*, Las Vegas, NV, 2007.
[18] H. Van Der Auweraer and J. Leuridan, "Multiple Input Orthogonal Polynomial Parameter Estimation," *Mechanical Systems and Signal Processing,* vol. 1, pp. 259-272, 1987.
[19] E. Balmes, "Integration of Existing Methods and User Knowledge in a MIMO Identification Algorithm for Structures with High Modal Densities," in *11th International Modal Analysis Conference (IMAC XI)*, Kissimmee, Florida, 1993, pp. 613-619.
[20] M. S. Allen, "Global and Multi-Input-Multi-Output (MIMO) Extensions of the Algorithm of Mode Isolation (AMI)," in *George W. Woodruff School of Mechanical Engineering* Atlanta, Georgia: Georgia Institute of Technology, 2005, p. 129.
[21] R. J. Allemang and D. L. Brown, "A Unified Matrix Polynomial Approach to Modal Identification," *Journal of Sound and Vibration,* vol. 211, pp. 301-322, 1998.
[22] M. S. Allen and J. H. Ginsberg, "A Global, Single-Input-Multi-Output (SIMO) Implementation of The Algorithm of Mode Isolation and Applications to Analytical and Experimental Data," *Mechanical Systems and Signal Processing,* vol. 20, pp. 1090–1111, 2006.
[23] P. Guillaume, P. Verboven, S. Vanlanduit, H. Van Der Auweraer, and B. Peeters, "A Poly-Reference Implementation of the Least-Squares Complex Frequency-Domain Estimator," in *International Modal Analysis Conference (IMAC XXI)*, Kissimmee, Florida, 2003.
[24] S. Rothberg, "Numerical simulation of speckle noise in laser vibrometry," *Applied Optics,* vol. 45, pp. 4523-33, 2006.
[25] S. J. Rothberg and B. J. Halkon, "Laser vibrometry meets laser speckle," Ancona, Italy, 2004, pp. 280-91.
[26] T. G. Carne, D. Todd Griffith, and M. E. Casias, "Support conditions for experimental modal analysis," *Sound and Vibration,* vol. 41, pp. 10-16, 2007.
[27] M. Rades and D. J. Ewins, "MIFs and MACs in Modal Analysis," in *20th International Modal Analysis Conference (IMAC-20)*, Los Angeles, CA, 2002, pp. 771-778.

[28] M. S. Allen and J. H. Ginsberg, "A linear least-squares version of the algorithm of mode isolation for identifying modal properties. Part II: Application and Assessment," *Journal of the Acoustical Society of America (JASA),* vol. 116, pp. 908-915, 2004.

[29] J. H. Ginsberg and M. S. Allen, "A linear least-squares version of the algorithm of mode isolation for identifying modal properties. Part I: Conceptual development," *Journal of the Acoustical Society of America (JASA),* vol. 116, pp. 900-907, 2004.

[30] M. S. Allen and J. H. Ginsberg, "Global, Hybrid, MIMO Implementation of the Algorithm of Mode Isolation," in *23rd International Modal Analysis Conference (IMAC XXIII)*, Orlando, Florida, 2005.

[31] M. S. Allen and J. H. Ginsberg, "Modal Identification of the Z24 Bridge Using MIMO-AMI," in *23rd International Modal Analysis Conference (IMAC XXIII)*, Orlando, Florida, 2005.

[32] J. H. Ginsberg, *Mechanical and Structural Vibrations*, First ed. New York: John Wiley and Sons, 2001.