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RESTORING FORCE SURFACE ANALYSIS OF NONLINEAR VIBRATION DATA FROM MICRO-CANTILEVER BEAMS

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ABSTRACT

The responses of micro-cantilever beams, with lengths ranging from 100-1500 microns, have been found to exhibit nonlinear dynamic characteristics at very low vibration amplitudes and in near vacuum. This work seeks to find a functional form for the nonlinear forces acting on the beams in order to aide in identifying their cause. In this paper, the restoring force surface method is used to non-parametrically identify the nonlinear forces acting on a 200 micron long beam. The beam response to sinusoidal excitation contains as many as 19 significant harmonics within the measurement bandwidth. The nonlinear forces on the beam are found to be oscillatory and to depend on the beam velocity. A piecewise linear curve is fit to the response in order to more easily compare the restoring forces obtained at various amplitudes. The analysis illustrates the utility of the restoring force surface method on a system with complex and highly nonlinear forces.

INTRODUCTION

The responses of micro-cantilever beams to base excitation have been found to exhibit significant nonlinearity, even when the tip displacement is less than 0.5% of the beam length. The beams were manufactured using the Sandia Ultra-planar Multi-level MEMS Technology (SUMMiT^{TM}) fabrication process at Sandia National Laboratories. This process is used to make various Micro-Electro-Mechanical Systems (MEMS), so the nonlinearity present in the beams may also be present in some MEMS devices made with the same process. This work is part of an effort to understand and model the nonlinearity in these types of systems.

A number of tests have been performed as part of this effort. The tip velocity of the beams is measured in near vacuum (~10 millitorr pressure) using a Laser Doppler Vibrometer (LDV) while the base of the beams is excited with a piezoelectric actuator. A second LDV is used to record the base velocity simultaneously. The bandwidth of the base excitation was limited so that only the first mode of the beam responded significantly.

The Restoring Force Surface (RFS) method by Masri and Caughey [1] was applied to the micro-beam base and response data. An attractive feature of the RFS method is that it does not require a parametric form for the nonlinearity a priori. This was invaluable in the present study because the cause of this nonlinearity is not known and tests with sinusoidal excitation show that many harmonics are excited, so traditional methods that rely on capturing nonlinearity with a polynomial series would be inefficient and possibly suffer from numerical ill-conditioning. Furthermore, because it is non-parametric, the RFS method allows one to present the data as traces of force versus displacement that can be easily interpreted. One drawback to the RFS method is that it is limited to single-degree of freedom or lumped parameter multi-degree of freedom systems [2]. This method determines the restoring force acting on a mass at various positions and velocities, which can then be examined or fit to a standard form. Kerschen *et al.* [3] recently presented an excellent summary of these and many other nonlinear system identification algorithms.

The restoring force found from sinusoidally excited tests on a single beam are reported, and shown to display a highly complicated topology as a function of position and velocity. The limited data available near zero velocity suggest that the force versus displacement relationship is linear, while at nonzero velocity the restoring force is highly oscillatory and causes the beam tip to accelerate and decelerate in a sinusoidal manner.

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NOMENCLATURE

w_L = Relative tip deflection, [m]
y = Absolute tip displacement, [m]
e_b = Base displacement, [m]
y_0 = Initial gap between beam and base, [m]
ω = Frequency, [rad/s]
ω_n = Natural frequency, [rad/s]
ζ = Damping ratio, [unitless]
f_{tot} = Total restoring forces [N]
f_{nl} = Nonlinear part of restoring forces [N]
m = Effective Mass [kg]

PROCEDURE

Figure shows a schematic of the system under study consisting of a cantilever beam attached to a moving base. The beam is excited around its first resonance, so assuming that modal coupling is negligible (as will be illustrated subsequently), it can be approximated as an oscillator subject to base excitation. The oscillator is assumed to be connected to its base by a nonlinear element resulting in the following differential equation

$$m\,\ddot{y} = -f_{tot}\left(\dot{w}_L, w_L\right) \qquad (1),$$

where y is the absolute displacement of the tip of the beam, w_L is the displacement of the beam tip relative to the substrate

$$w_L = y - e_b \qquad (2),$$

and m is the effective mass of the cantilever.

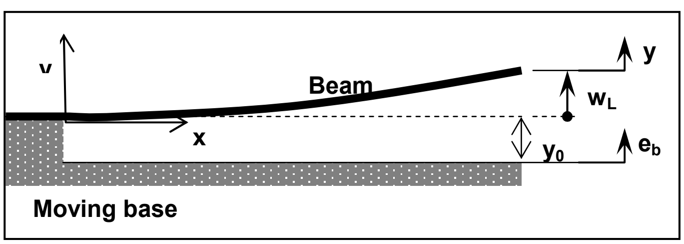

Figure 1: Schematic of the system under study.

Equation (1) simply states Newton's second law for this system; the effective mass of the cantilever times its acceleration is equal to the applied forces on the cantilever tip. This is the basic idea behind the restoring force surface method; the measured acceleration is proportional to the forces applied to a single-degree of freedom system. The applied forces are typically assumed to be some nonlinear function of the relative displacement and relative velocity between the cantilever and the substrate. If the displacement and velocity can also be measured, then the functional relationship between the restoring forces and the displacement and velocity can be evaluated. From this point forward the mass will be taken to be unity since our objective is to obtain the functional form of the nonlinear forces. The term "restoring force" will be applied to the unscaled acceleration since this is equal to the restoring forces after scaling by the constant unknown mass. A number of researchers have discussed methods for identifying the system mass [2, 4].

As an example, consider a linear system whose natural frequency and damping ratio are 82.5 kHz and 0.0001 respectively. The restoring force surface for such a system (scaled by unit mass) is displayed in Figure 2. As expected, the RFS is a plane whose slope is the natural frequency squared in the w_L direction and twice the damping ratio times the natural frequency in the $\dot{w}_L$ direction. Notice that the slope in the $\dot{w}_L$ direction is very small for a lightly damped system.

The linear terms can be extracted from the restoring force f_{tot} so that the unknown nonlinear function contains only nonlinear terms f_{nl}

$$\ddot{y} = -2\zeta\omega_n\dot{w}_L - \omega_n^{\,2}w_L - f_{nl}\left(\dot{w}_L, w_L\right)/m \qquad (3),$$

where ω_n and ζ are the natural frequency and damping ratio of the system respectively. It is sometimes useful to examine only this nonlinear part of the restoring forces, especially when the measured data is weakly nonlinear.

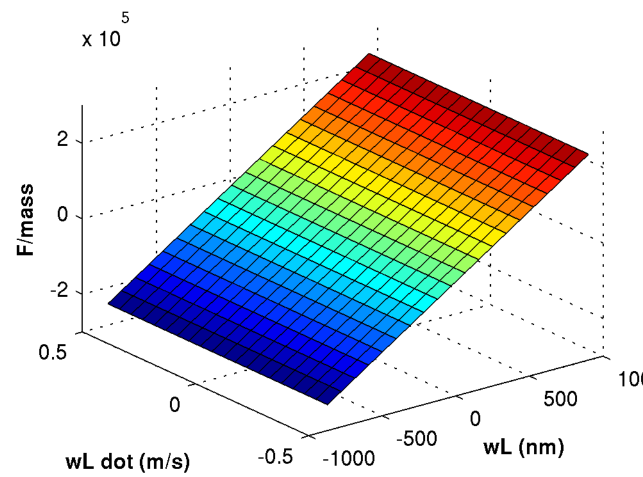

Figure 2: Restoring force surface for a linear system.

Equation (1) reveals that the restoring force as a function of displacement and velocity can be easily found if the signals y, w_L and $\dot{w}_L$ in Eq. (1) can be measured or estimated. The most common approach is to measure one of the signals and then integrate or differentiate numerically to obtain the others. This approach was adopted in this work. One must take great care, however, to assure that the numerical differentiation or integration schemes employed are consistent with the sampling assumption used when collecting the data. For example, most data acquisition systems measure a signal over a certain bandwidth and use anti-aliasing filters to assure that the measured signal is effectively band limited. However, many numerical differentiation techniques are not consistent with this assumption because they implicitly assume non-band limited signals. For example, forward difference differentiation and trapezoidal integration both assume that a signal can be linearly interpolated between samples, and such a signal is not truly band limited. Schoukens *et al.* provide a good discussion of the implications of band-limited and piecewise constant excitation signals in system identification [5].

Although the RFS method allows for non-parametric identification of nonlinear systems, researchers commonly use the method to fit polynomial models to experimental data [2]. Polynomial models may require many terms to describe complex nonlinearities. This is especially troublesome because high order polynomial models are difficult to fit to measured response data due to numerical ill conditioning. Furthermore, the polynomial terms can have vastly different magnitudes, so it can be difficult to establish the significance of each. Duym *et al.* [4] presented a computationally efficient method for determining the local mean and slope of a restoring force surface from data that may help to address this problem in many applications. Unfortunately, the method of Duym *et al.* does not assure that the locally determined restoring forces can be stitched together into a continuous surface, so they may be of limited value for difficult applications. The work presented here utilized a variation on the method of Duym *et al.* that fits a true piecewise linear approximation to the restoring force. The measured data will also be presented in scatter plots to aid in evaluating the force displacement and force velocity relationships and to visualize the relative importance of experimental scatter and nonlinearity. The restoring force surface method is discussed in more detail in [1-4, 6] and their references.

EXPERIMENTAL SETUP

An array of micro cantilever beams was created using Sandia National Laboratories' SUMMiT^{TM} Process on a silicon wafer. This work is concerned with a beam that had nominal length of 200 μm, width of 10 μm and height of 2.5 μm. It was fixed to the substrate at one end and free at the other. The beam was constructed by depositing two poly-silicon layers of thicknesses 1.5 μm and 1.0 μm over a 2 μm layer of sacrificial oxide and then removing the oxide. Figure 3 shows an optical microscope image of an array of beams, including the beam that was tested. Additional poly-silicon layers were placed over the root of the array of beams, as seen in Figure 3. The beams were placed in a chamber fastened to a piezoelectric actuator. The assembly was fastened to an aluminum block that served as a seismic mass. The air in the chamber was evacuated resulting in a test pressure of about 10 millitorr. Laser measurements were obtained by imaging through a quartz window.

The base of the cantilever was driven by applying an 82 kHz, sinusoidal voltage to the piezoelectric actuator with amplitudes ranging from 0-20 Volts, corresponding to near-resonant excitation. The time responses of both the beam tip and substrate were recorded, sampled at 5 MHz. The laser hardware includes a low-pass anti-aliasing filter with cutoff frequency of 1.5 MHz.

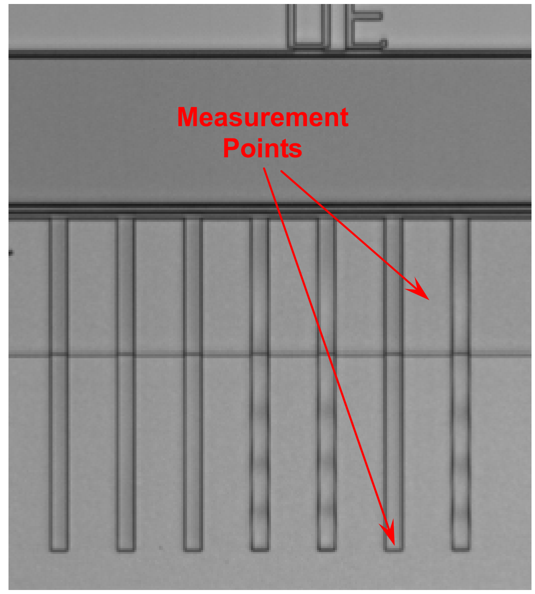

Figure 3: Optical microscope image of micro-cantilever beams.

The base velocity and the velocity of the tip of the beam were both measured using a Polytec Laser Doppler Vibrometer (LDV) focused through a Mitutoyo optical microscope with a 10X objective lens. The velocities were integrated and differentiated using the technique described by Smallwood [7], which is consistent with the assumption of band limited measurements. Eq. (1) shows that the restoring force is equal to the derivative of the measured tip velocity $\dot{y}$. It will be evaluated as a function of the relative tip displacement w_L and its derivative $\dot{w}_L$. The relative displacement and relative displacement velocity w_L and $\dot{w}_L$ were found by taking the difference between the integrated and measured tip and base velocities respectively. All of the signals were high-pass filtered with an 8th order Butterworth filter with a cutoff frequency of 40 kHz to eliminate the spurious drift caused by the laser.

RESULTS

The autospectra of the base and tip velocities of this beam for one excitation are displayed in Figure 4. Both autospectra are dominated by an 82 kHz sinusoid. The tip autospectrum shows multiple harmonics whose magnitudes are 40-100 dB below the fundamental. (The autospectrum is squared quantity so the velocity of these harmonics would be as much as 10% or as little as 0.3% of the velocity of the primary 82 kHz sinusoid.) Similar results were obtained at other excitation amplitudes, although the bandwidth of the harmonics was observed to increase with increasing excitation amplitude.

Other peaks are also evident at frequencies that are not integer multiples of the excitation frequency. For example, there are peaks at 520 kHz and 1450 kHz. If the beam were an ideal Euler-Bernouli cantilever with constant cross sectional properties and a first mode at 82.5 kHz, one would expect the modal frequencies of the second and third modes to occur at 517 and 1447 kHz [8]. The peaks at 520 and 1450 kHz are 100 dB or more below the response at the excitation frequency, so their effect on the response should be small. Peaks at these frequencies were also observed at other excitation amplitudes. They were likewise small at all other excitation amplitudes except at 140 nm and 300 nm tip amplitude. It was also noted that the restoring force surface data at 140 and 300 nm tip amplitudes had the largest variability with respect to w_L.

Finally, one should note that the beam is very lightly damped with a Q of about 5000, or damping ratio of ~0.01%, so the base motion makes only a small contribution to the restoring force in Eq. (1).

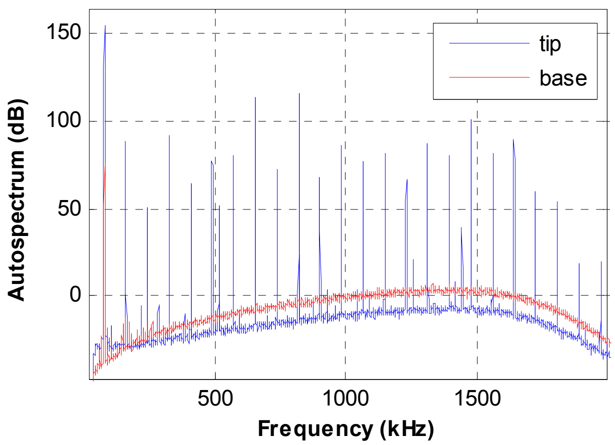

Figure 4: Autospectra of base and tip velocities for 13.3V excitation, 590nm peak tip displacement.

The restoring force, found using Eq. (1), can be evaluated at any pair of values $(w_L, \dot{w}_L)$ where the response has been measured. Figure 5 shows the points in the $(w_L, \dot{w}_L)$ plane through which the response passes during the 500 samples for various excitation amplitudes. The amplitudes listed are the peak tip displacements w_L observed in nanometers, and correspond to voltage amplitudes of 3.3V, 6.6V, 10V, 13.3V, 16.6V and 20V. Multiple data sets are shown for most of the amplitudes, illustrating the level of repeatability observed in the measurements. The phase portraits at amplitudes below 600nm are all quite repeatable, whereas the different trials for peak amplitudes above 600nm show significant variation. All of the phase portraits differ from the pure elliptical motion expected for a linear time invariant system. This deviation takes the form of ripples in the phase portrait at the points where the magnitude of the velocity is largest, although the velocity at which the rippling begins changes with differing excitation amplitude.

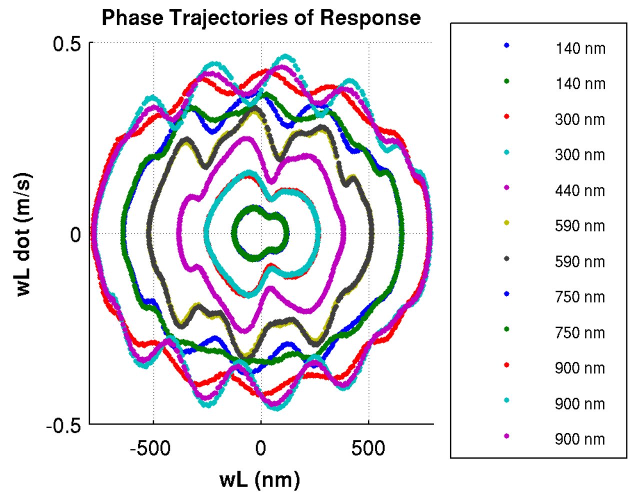

Figure 5: Phase portrait of beam responses at various tip displacement amplitudes for sinusoidal excitation at 82 kHz.

The restoring force surface values for 2500 samples of each response are displayed in Figure 6, with the value of the restoring force indicated by the color of the circles. The restoring force at the highest amplitudes is clearly multi-valued, suggesting that the force is altered by effects other than the tip displacement and velocity. At moderately high amplitudes the restoring force changes sign a number of times as the cantilever tip executes one vibration cycle. Once can also note a clear linear trend in the data for increasing displacement w_L, suggestive of an underlying linear stiffness. A clearer view of the restoring force can be obtained by observing the restoring force as a function of displacement only. In doing so, it was found that the restoring force was significantly different for positive and negative $\dot{w}_L$ at some amplitudes, so these will be considered separately.

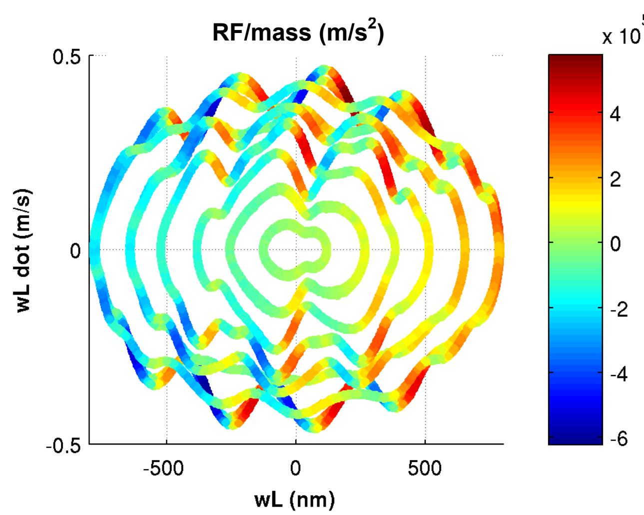

Figure 6: Scatter-plot of restoring force surface

Figures 7 and 8 show the data for $\dot{w}_L > 0$ from two measurements and a piecewise linear fit to it for 140 and 590 nm tip amplitudes respectively. The two data sets (shown in blue and green) are highly consistent for both 140 and 590 nm tip amplitude, and the piecewise linear approximation fits them well. The data at 140 nm amplitude shows considerably more scatter, yet the piecewise linear fit follows the mean trend of the data well. These data sets are representative of the best and worse agreement found below 600 nm.

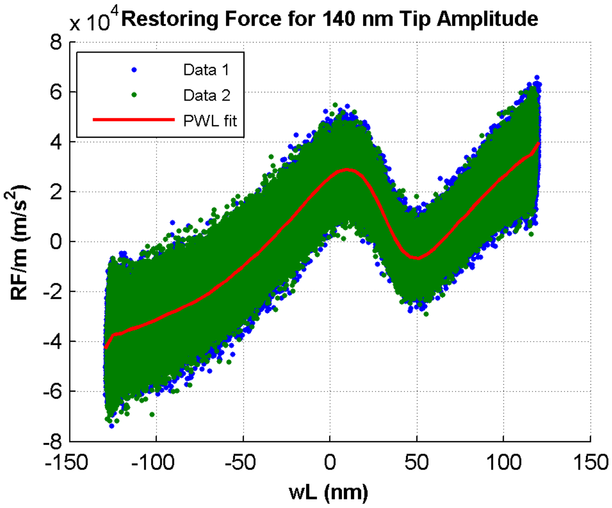

Figure 7: Restoring force data with $\dot{w}_L > 0$ versus w_L for two data sets and piecewise linear fit at 140 nm tip amplitude.

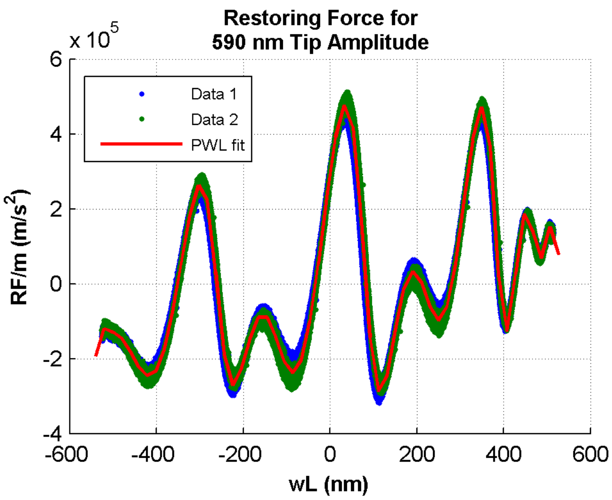

Figure 8: Restoring force data with $\dot{w}_L > 0$ versus w_L for two data sets and piecewise linear fit at 590 nm tip amplitude.

Figure 9 shows the piecewise linear fits to the restoring force found at various tip amplitudes for both positive and negative velocities, shown with solid and dashed lines respectively. The curves are highly oscillatory, and the level of oscillation increases with amplitude. By comparing Figures 5 and 9, one can note that increasing tip amplitude implies increasing velocity throughout most of the response.

It is tempting to think of the lines in Figure 9 as static force versus displacement curves, yet one must bear in mind that the velocity is nonzero in each of them. Hence, the negative slope observed near the center of many of the curves does not imply that the structure has negative stiffness, but only that the net force acting on the moving beam tip is decreasing at that point in the cycle.

One can approximate the quasi-static force displacement curve for the cantilever by extracting from Figure 6 only the data for which the absolute value of the velocity is small. It is important to recall that the data was high-pass filtered to remove spurious drift, so the filtered response data does not accurately represent the low-frequency response. As a result, this approach will only be successful if the low frequency dynamics of the nonlinear response data are negligible, for example, if the mean of the true nonlinear response is actually near zero. Figure 10 shows the result, considering all of the data for which the tip speed $|\dot{w}_L|$ was less than 0.00005 m/s. A least squares fit to the linear trend in the data is also displayed. There is a fair amount of scatter in the data, yet the overall trend appears to be approximately linear. The slope of the linear trend can be used to compute the linear natural frequency, yielding a value of 81.5 kHz. This is somewhat lower than the observed resonance frequency of 82.5 kHz.

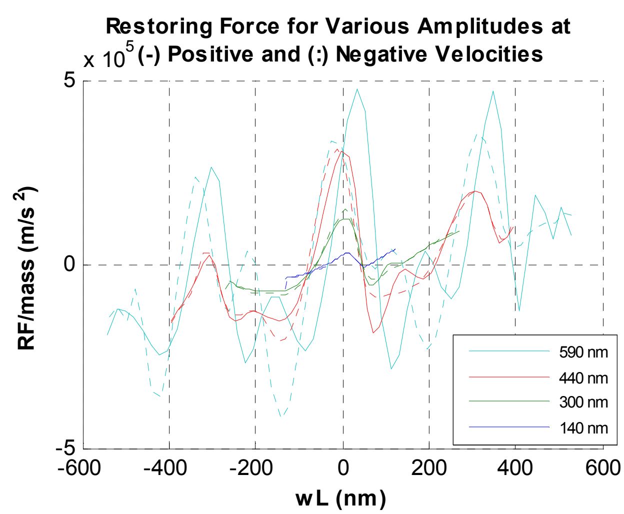

Figure 9: Piecewise linear fits to restoring force data versus w_L with $\dot{w}_L > 0$ (solid lines) and $\dot{w}_L < 0$ (dashed lines).

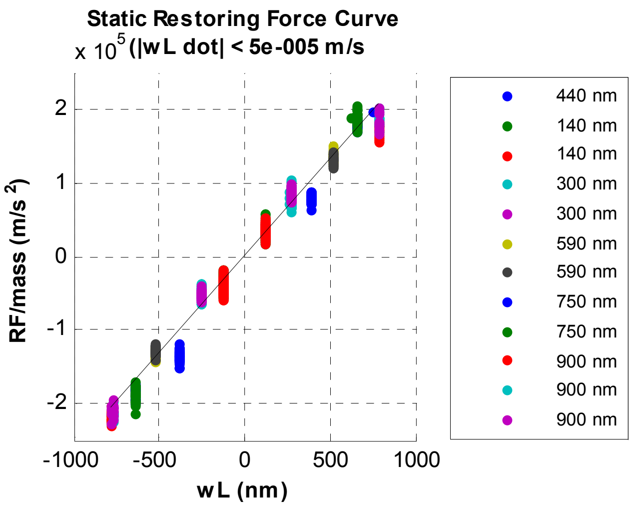

Figure 10: Quasi-static Force displacement curve obtained from the data at small velocity from multiple data sets and linear fit (black line)

DISCUSSION

The autospectra show that the beam response is clearly nonlinear, and suggest that the nonlinearity is responsible for as much as 10% of the response. On the other hand, Figure 9 shows that the magnitude of the nonlinear forces acting on the cantilever are much larger than the linear forces at some points in the cycle. (The linear force is shown in Figure 10 where the scale is 2/5 that in Figure 9.) The restoring forces presented in Figures 6 and 9 are not amenable to any simple force versus displacement model. Indeed, one can deduce from the autospectrum in Figure 5 that more than 19 terms would be required to describe the restoring force using a polynomial series. This suggests that one would need a high order NARMAX model to capture the system's nonlinearity, or a large number of pseudo-inputs if a Reverse Path [9] method were used. The Restoring force method proves invaluable in this application.

The restoring force found at 140 nm tip amplitude, shown in Figure 7, shows considerable scatter. It was also noted that the peaks in the autospectra in the vicinity of the 2nd and 3rd natural frequencies of the beam were considerable relative at this amplitude. The presence of the 2nd and 3rd modes in the response contaminates the restoring force, yet since these modes frequencies are not integer multiples of the first natural frequency or the excitation frequency, their contributions average out over the time record. These modes appear to have had little effect on the data at higher excitation amplitudes.

The autospectra in Figure 5 shows that the beam responds out beyond the 1.5 MHz limit of the laser decoder. Harmonics beyond the 19th may contain important information about the restoring force surface that was not captured. The harmonics in the autospectra for small tip displacement amplitudes (140, 300 and 440 nm) had decayed significantly at 1.5 MHz, suggesting that the restoring force should have been adequately captured at these amplitudes. On the other hand, the last few measured harmonics for the higher amplitude responses were significant, suggesting that the measurement bandwidth may have been insufficient for them.

It seems that one important clue to the cause of the nonlinearity is its velocity dependence. Figure 10 suggests that the force-displacement relationship is linear at zero velocity. On the other hand, Figure 6 shows that for any given amplitude, the response appears to be somewhat linear until a certain tip speed is reached, after which the response becomes erratic until the speed again falls below that threshold. The oscillatory nature of the response is reminiscent of a stick-slip phenomenon, yet the measurements never indicate a zero velocity for the tip. It is possible that the phase portraits are not captured completely in the measurements due to the limited bandwidth of the laser vibrometer, so the velocity could in fact be smaller than indicated in Figures 5 and 6, and may even reach zero.

It was also noted that the restoring force surface is multi-valued at high amplitudes. This might also indicate a stick-slip type phenomenon, since such a phenomenon cannot be captured by a single valued restoring force surface. Hysteretic phenomena such as stick-slip yield a multi-valued restoring force surface, since the force in such a system depends not only on displacement and velocity, but also on unmeasured slip states or on response history. Considering the layer-wise manufacturing process for these cantilevers, it certainly seems reasonable that the structure may contain regions where the interlayer adhesion is poor that could act as sliding surfaces. There might also be regions near the root where some of the sacrificial oxide remains that the poly-silicon beam may slide against. The phenomenon observed here might also be one of pure sliding where a stuck type state is reached at low, yet nonzero velocity.

This paper presents test data from only one beam. The authors have observed qualitatively similar responses from other beams of lengths ranging from 100 - 1500 microns, widths ranging from 10 to 30 microns and similar thicknesses (1.5-2.5 microns).

CONCLUSIONS

Experimental measurements from a micro-cantilever beam were presented that exhibit a complicated, velocity-dependent nonlinearity. The beam was tested in vacuum and was excited by applying sinusoidal excitation to its base of varying amplitudes. The restoring force surface (RFS) method was used to characterize the nonlinearity because it allows for non-parametric analysis of nonlinear response data. The RFS method provided powerful insight into the dynamics of this system, and the results suggest that other nonlinear identification methods, such as NARMAX, Higher Order Spectra, or Reverse Path type methods [3] would have proven exceedingly difficult to apply since they rely on a polynomial representation for the nonlinearity.

It appears that the nonlinearity observed is caused by sliding friction, since it is most severe at high beam velocities. Efforts are underway to validate this hypothesis and to develop a mathematical model that recreates this behavior.

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