Comparison of FRF and Modal Methods for Combining Experimental and Analytical Substructures

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Outline

- Motivation
- Modal Coupling vs. FRF Coupling

- Component Mode Synthesis Theory
- Methods for connecting subsystems
  - Connection Point Method (CPT)
  - Modal Constraint for Fixture and Subsystem (MCFS)
- Experimental Results

  - Rigid Fixture Model with CPT constraint
  - Elastic Fixture Model with CPT constraint
  - Elastic Fixture with MCFS constraint
- Conclusions

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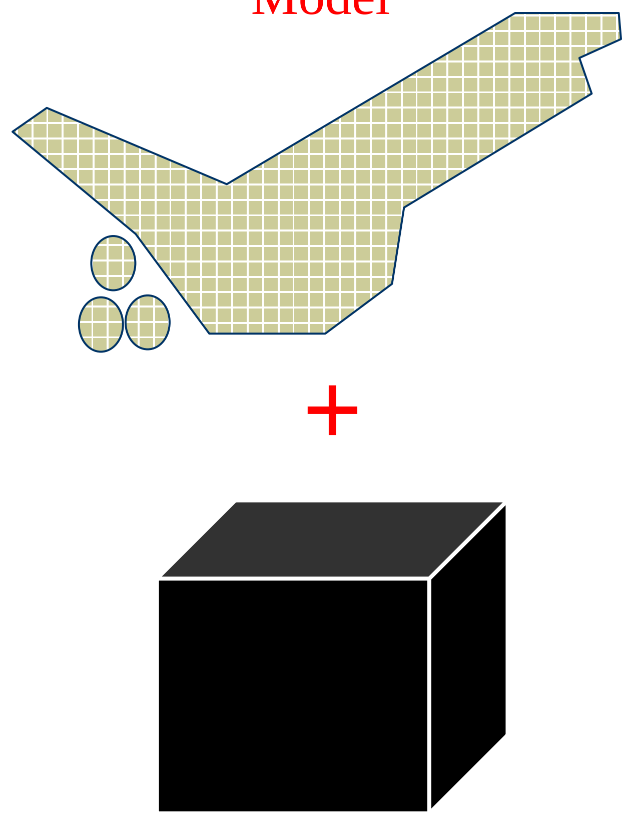
Motivation

- Subcomponents are often designed by a number of independent groups that do not have the information or the resources to model the macro system.
- In other applications some particular components may be too difficult to model analytically with the required precision.

Analytical Model



Experimental Model

Modal vs. FRF Coupling

- Two distinct approaches to substructure coupling exist:

- Component Mode Synthesis or Modal Coupling:
  - Modal models for substructures combined to find an approximate modal model for the total system.
- Requires modal parameter estimation to reduce measurements to a modal system model.
  - Measurements must be sufficient to obtain reasonably accurate estimates of each important modal frequency and mode shape at the connection points.
- Results can sometimes be understood by appealing to well-established Ritz Theory.
- Disadvantage: It is not always easy to identify an adequate modal model from experimental measurements.

- FRF Based Admittance or Impedance Coupling:
  - Response measurements (FRF matrices) for substructures combined to find FRFs for total system.
- Can be performed on raw FRFs (even if modal parameter estimation is not feasible.)
  - However, all connection point FRFs must be measured if MPE is not employed.
    - $6*N_c \times 6*N_c$ set of FRFs!
  - Numerical ill-conditioning may present a formidable challenge.
  - Modal parameter estimation is very desirable to reduce measurement errors for lightly damped systems. [Imregun, Robb, Ewins IMAC-1987]

Component Mode Synthesis

- Given a set of modal parameters, one has a set of equations of motion (EOM) for a substructure:

$$[I]\{\ddot{\eta}\} + [\omega_r^2]\{\eta\} = [\Phi]^T\{F\}$$

$$\{y\} = [\Phi]\{\eta\}$$

  - $\{y\}$ is a vector of physical coordinates
  - $\{\eta\}$ are modal coordinates
  - $[\omega_r^2]$ is a diagonal matrix of natural frequencies squared
  - $[\Phi]$ is a matrix of mass-normalized mode vectors.
- Consider two independent structures A and B:

- Their EOM are….

Component Mode Synthesis (2)

$$\begin{bmatrix} [I]_A & 0 \\ 0 & [I]_B \end{bmatrix} \begin{Bmatrix} \{\ddot{\eta}\}_A \\ \{\ddot{\eta}\}_B \end{Bmatrix} + \begin{bmatrix} [\omega_r^2]_A & 0 \\ 0 & [\omega_r^2]_B \end{bmatrix} \begin{Bmatrix} \{\eta\}_A \\ \{\eta\}_B \end{Bmatrix} = \begin{Bmatrix} [\Phi]_A^T \{F\} \\ [\Phi]_B^T \{F\} \end{Bmatrix}$$

- Connect them by enforcing linear constraints:

$$(y_c)_A = (y_c)_B$$

- All DOF can be expressed in terms of an unconstrained set of generalized coordinates as

$$\begin{Bmatrix} \{\eta\}_A \\ \{\eta\}_B \end{Bmatrix} = [P] \begin{bmatrix} -[\hat{a}_c]^{-1} [\hat{a}_u] \\ [I] \end{bmatrix} \{\eta\}_u = [B]\{\eta\}_u$$

Paper includes a method that chooses constrained DOF so that this matrix is always invertible if such a choice exists.

- The equations of motion in unconstrained coordinates are then:

$$[\hat{M}]\{\ddot{\eta}\}_u + [\hat{K}]\{\eta\}_u = \{Q\}$$

$$[\hat{M}] = [B]^T \begin{bmatrix} [I]_A & 0 \\ 0 & [I]_B \end{bmatrix} [B]$$

$$[\hat{K}] = [B]^T \begin{bmatrix} [\omega_r^2]_A & 0 \\ 0 & [\omega_r^2]_B \end{bmatrix} [B]$$

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Component Mode Synthesis (3)

- One can now find the modes of the combined system, construct FRFs, etc… using these EOM.
- The mode shapes of the combined system are found in the physical coordinates of A and B.
  - The combined system mode shapes are linear combinations of the mode shapes of the subcomponents A and B.
  - One must assure that enough modes of both A and B are present so that these are a good approximation for the true modes of the combined system.

Simulation Example

- Case 1: Free-Free (FF) modes – modal parameters of each FF beam used to predict combined system response:

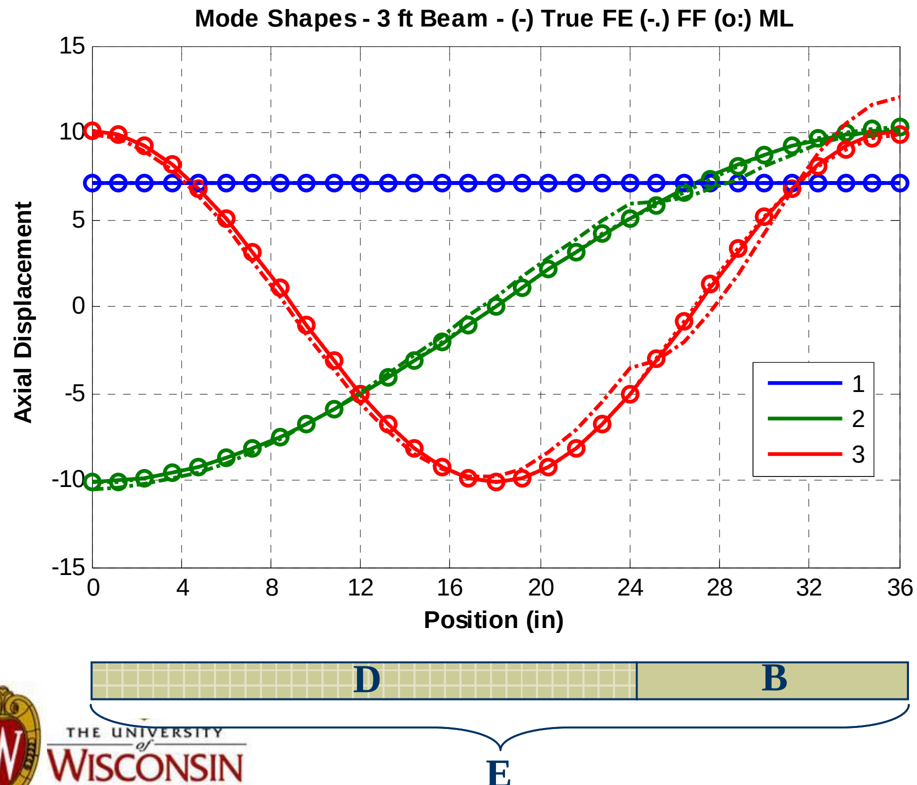
- Case 2: Mass-Loaded (ML) modes – parameters of modes with fixture on B.

- Cases simulated by joining only the modes of B below 10 kHz to the FE model for D
- The result is compared to the full FE solution for E.

Simulation Example: Axial Results

Natural Frequencies (Hz)

Mode	E-FEA	E-FF (free-free)	E-ML (mass-loaded)	Error (E-FF) (free-free)	Error (E-ML) (mass-loaded)
1	0.0	0.0	0.0	0.0%	0.0%
2	2824.8	2919.4	2829.7	3.3%	0.2%
3	5649.1	5860.6	5651.3	3.7%	0.0%
4	8472.5	8472.5	8678.5	0.0%	2.4%
5	11294.3	11758.9	12171.5	4.1%	7.8%



- Both approaches predict the axial modes in the frequency band with less than 5% error.
- The mass loaded modes predict most of the natural frequencies and mode shapes more accurately.

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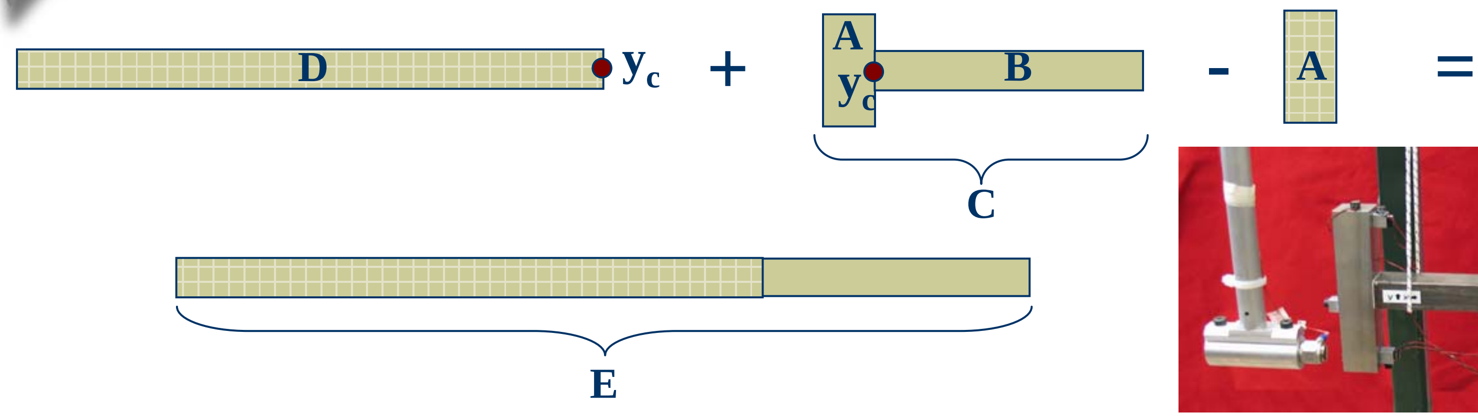
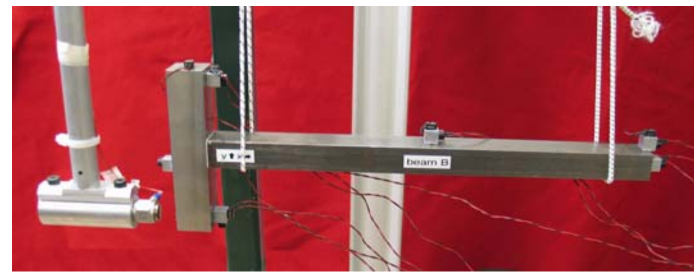
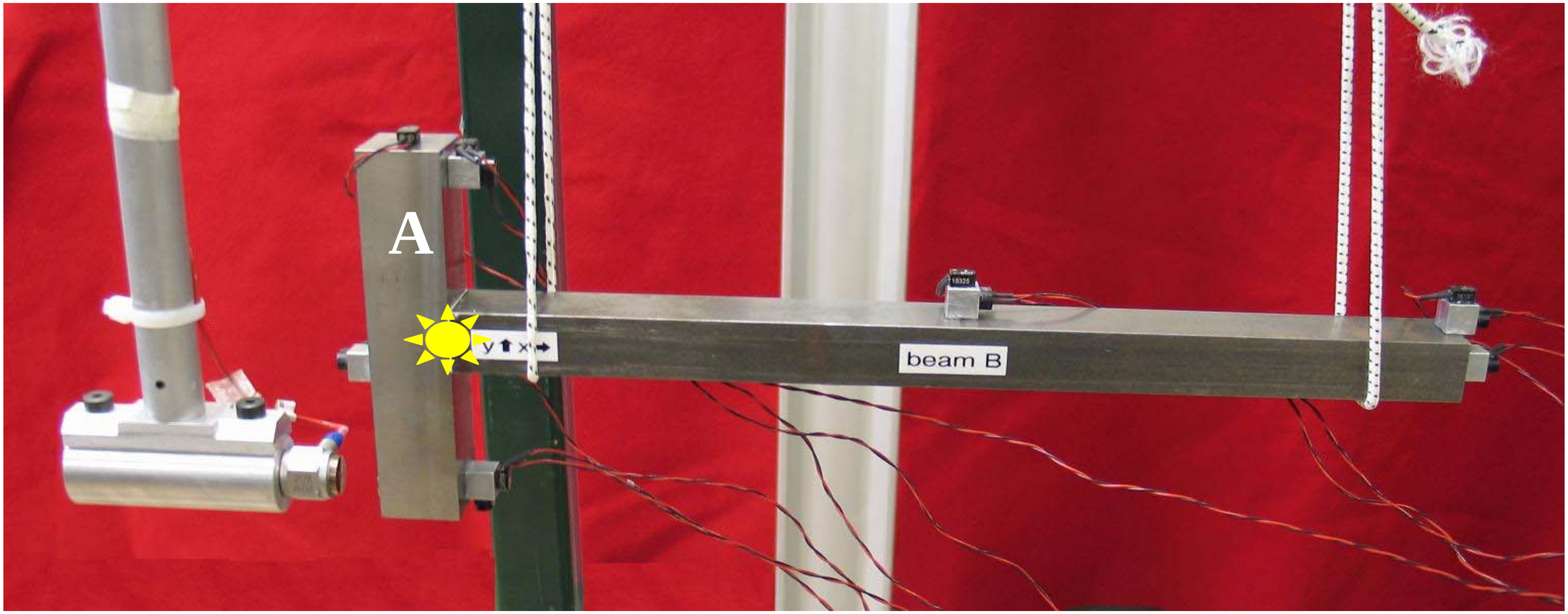
Test System





- Objective: Join an experimental model of beam B to an analytical model for beam D at point y_c.
  - Measurements are taken on system C consisting of beam B with fixture A attached.
  - Analytical model (Euler-Bernoulli) for fixture A removed from system C resulting in an experimental model for beam B.
    - Analytical model for fixture A
  - Beam B is then combined with an analytical model (tunned Euler-Bernoulli) for beam D to find the combined system Beam E.

Experimental Procedure



- Careful tests used to estimate the modal parameters of the C system at a number of points on fixture A.

Connection Methods – CPT and MCFS

- One cannot measure the response at the connection point directly, so it must be estimated from other measurements.
  - CPT Method: Connection point responses for the experimental system are estimated using a modal filter and constrained to the analytical fixture A and beam D:

$$\begin{Bmatrix} \{y^C\}_m \\ \{y^C\}_c \end{Bmatrix} \approx \begin{bmatrix} [\Phi^A{}_m] \\ [\Phi^A{}_c] \end{bmatrix} \{\eta^C\} \quad \rightarrow \quad \{y^C\}_c = [\Phi^A{}_c][\Phi^A{}_m]^\dagger \{y^C\}_m$$

  - MCFS Method: Constrain the modal DOF of the Fixture model to their approximation on C: (MCFS stands for Modal Constraint for Fixture and Subsystem)

$$\left.\begin{aligned} \{\eta^A\} &= [\Phi^A{}_m]^\dagger \{y^A\}_m \\ \{\eta^C\} &= [\Phi^A{}_m]^\dagger \{y^C\}_m \end{aligned}\right\} \quad \rightarrow \quad \{\eta^A\} = \{\eta^C\}$$

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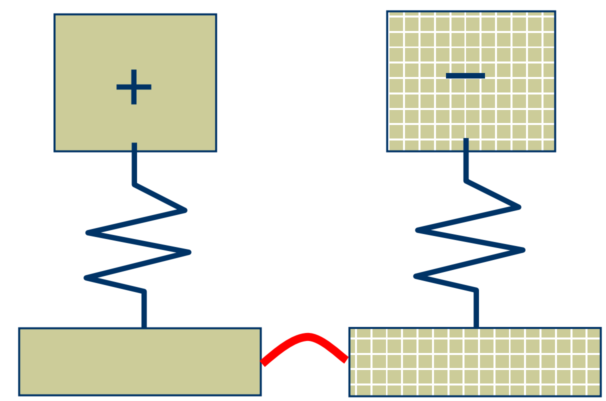
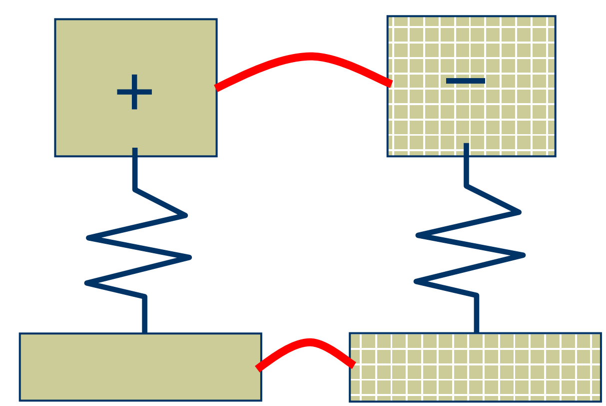
Connection Methods – Rationalization

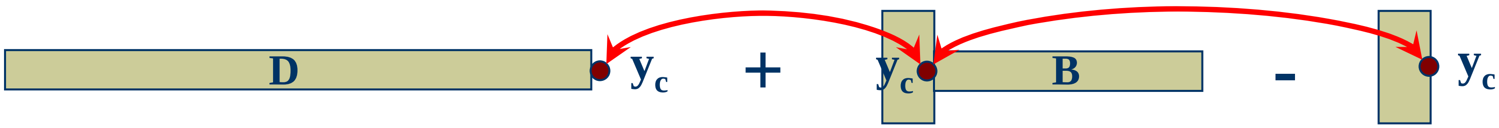
- CMS can be very sensitive to errors when removing a substructure from a system.
- In the problem considered here the fixture response is dominated by (4) modes.
- Requiring equal 2D motion at the connection point enforces only (3) constraints.
  - $4_{\text{DOF}} * 2_{\text{Systems}} - 3_{\text{Constraints}} = 5_{\text{Remaning DOF}}$
  - Two elastic modes remain in the system. One would hope that these have no effect on the response.
  - It might be preferable to remove these extra modes by adding constraints rather than simply hope that the fixture model is accurate enough so that they cancel.



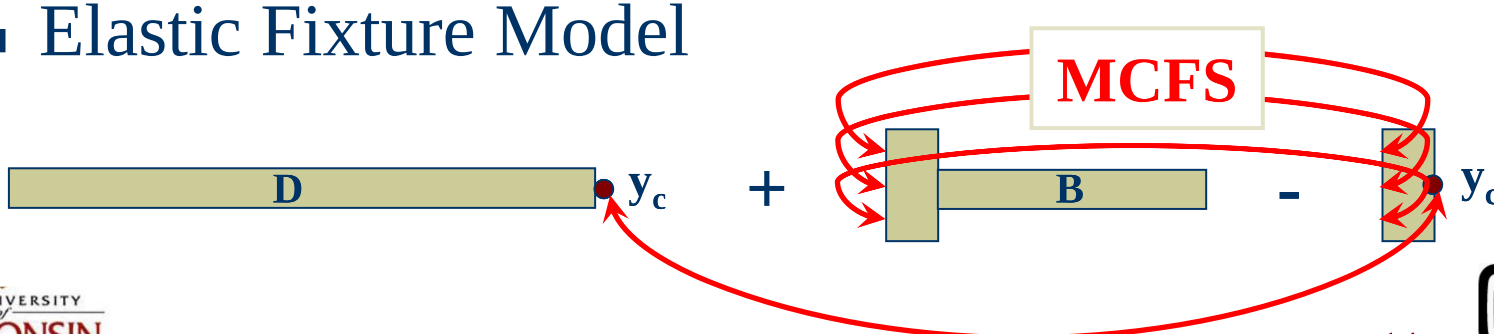


Cases Considered

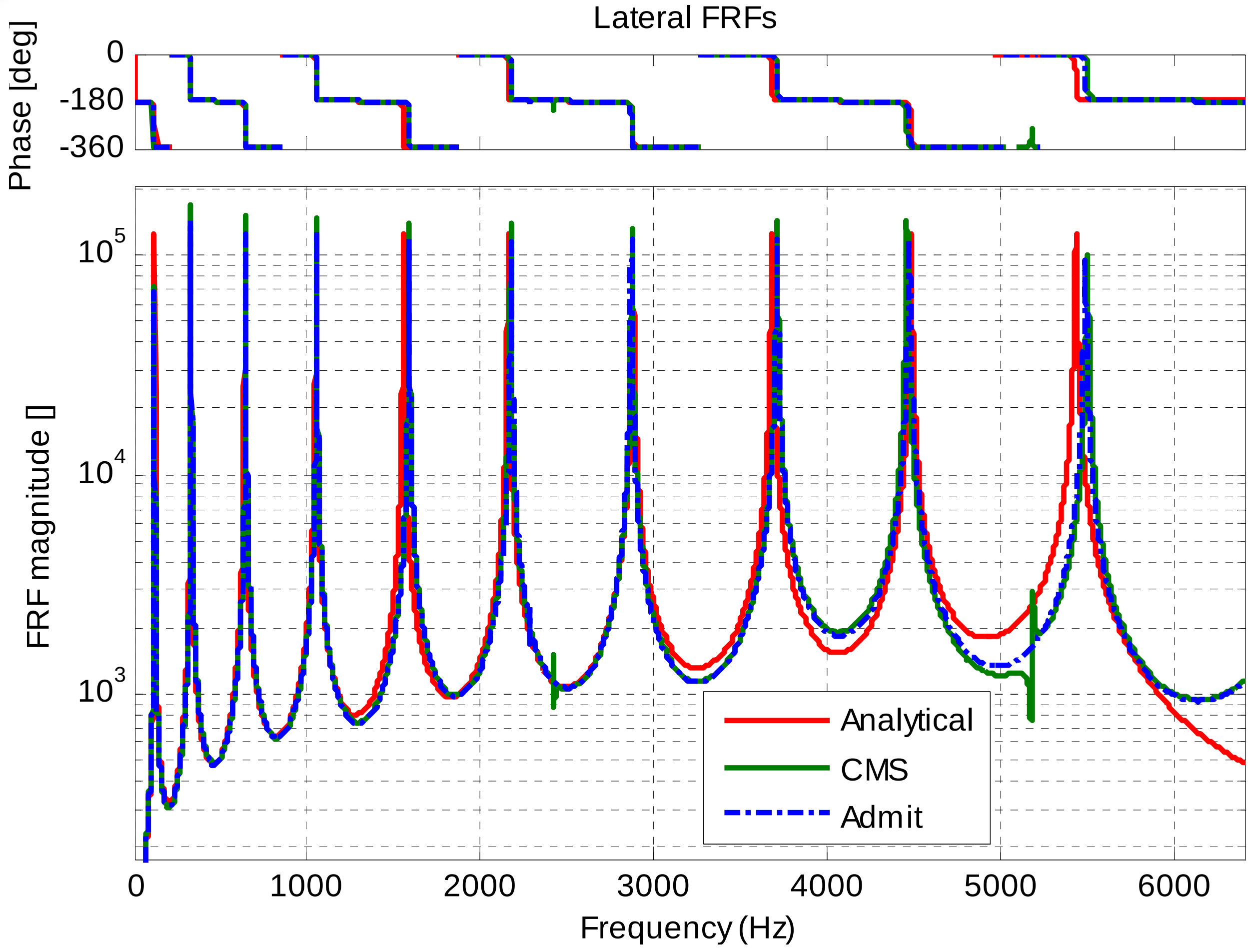
- Case 1: **CPT**: Models for A, C and D joined at the connection point.
  - Case 1a: Rigid Fixture Model
  - Case 1b: Elastic Fixture Model



- Case 2: **MCFS**: Models for A and C joined using MCFS method.
  - Elastic Fixture Model

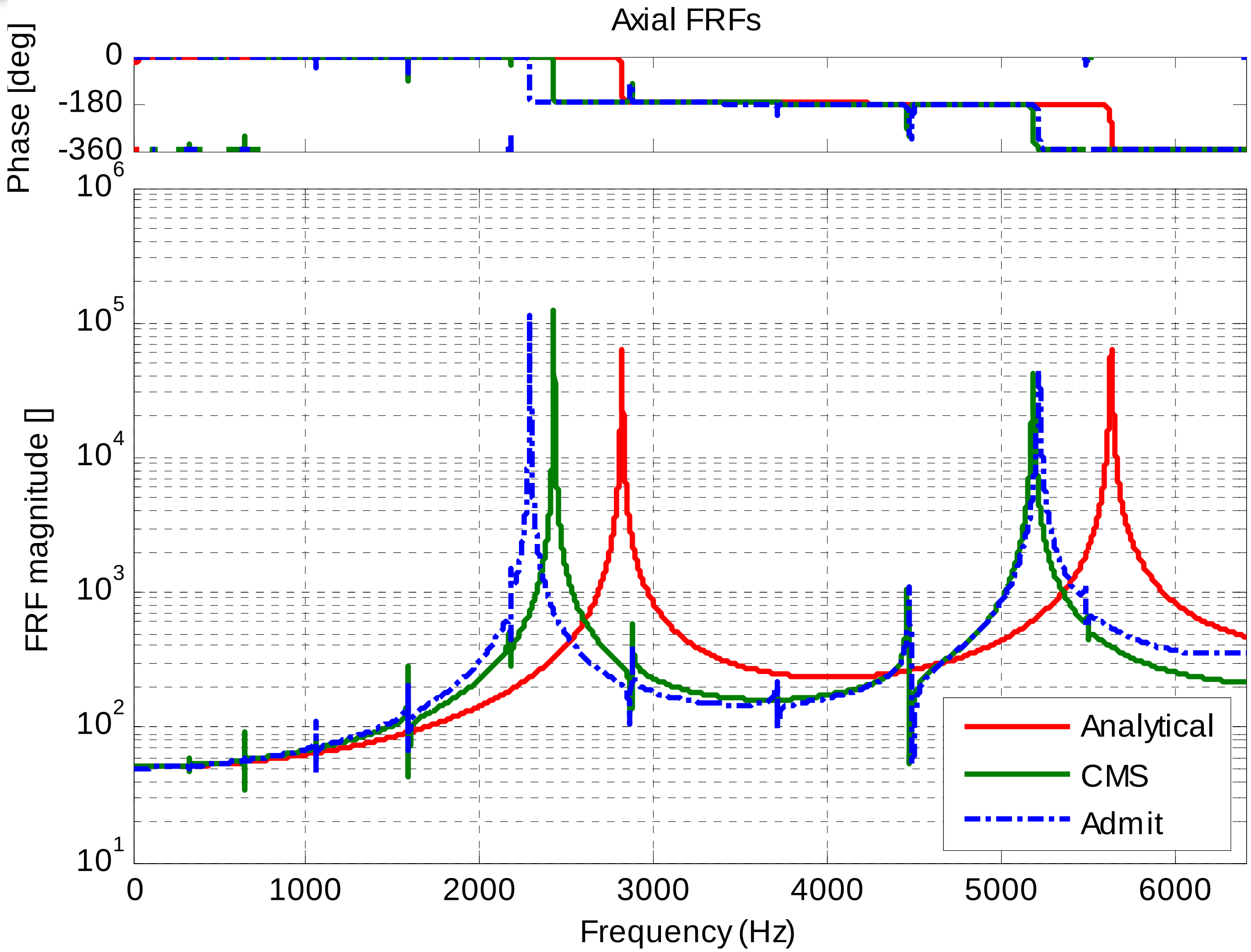


Case 1a: Rigid A, CPT



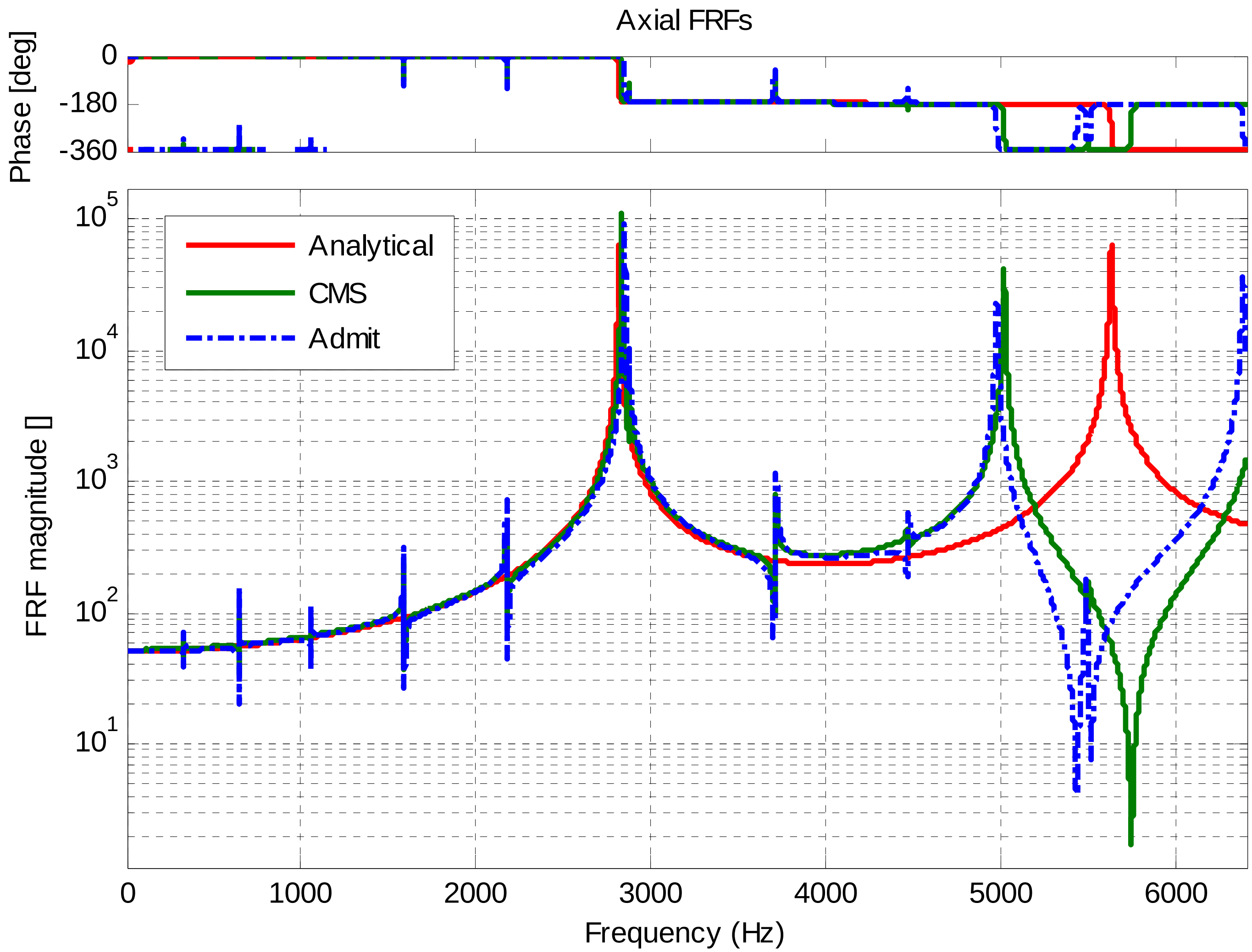
- Excellent results obtained in the lateral direction (Bending Modes).
- Both the CMS and FRF based Admittance procedures agree very well with the analytical model.
- The CMS result is slightly contaminated by the axial modes at 2400 and 5200 Hz.

Case 1a: Rigid A, CPT



- Admittance and CMS both under-predict the natural frequencies of the axial modes by 10% or more.
  - These errors are larger than one would expect due to modal truncation alone.
  - Rigid fixture model is not adequate.
- Both also over-predict the axial motion in the bending modes resulting in contamination at the bending natural frequencies.
  - Possibly due to small curve fitting errors or cross axis sensitivity.

Case 1b: Flexible A, CPT



- Both Admittance and CMS accurately predict the first axial frequency with the flexible fixture model.
- Both methods more severely under-predict the second axial mode.
- Both predict a spurious zero near 5500 Hz.
- The Lateral FRFs were similar to those shown previously.

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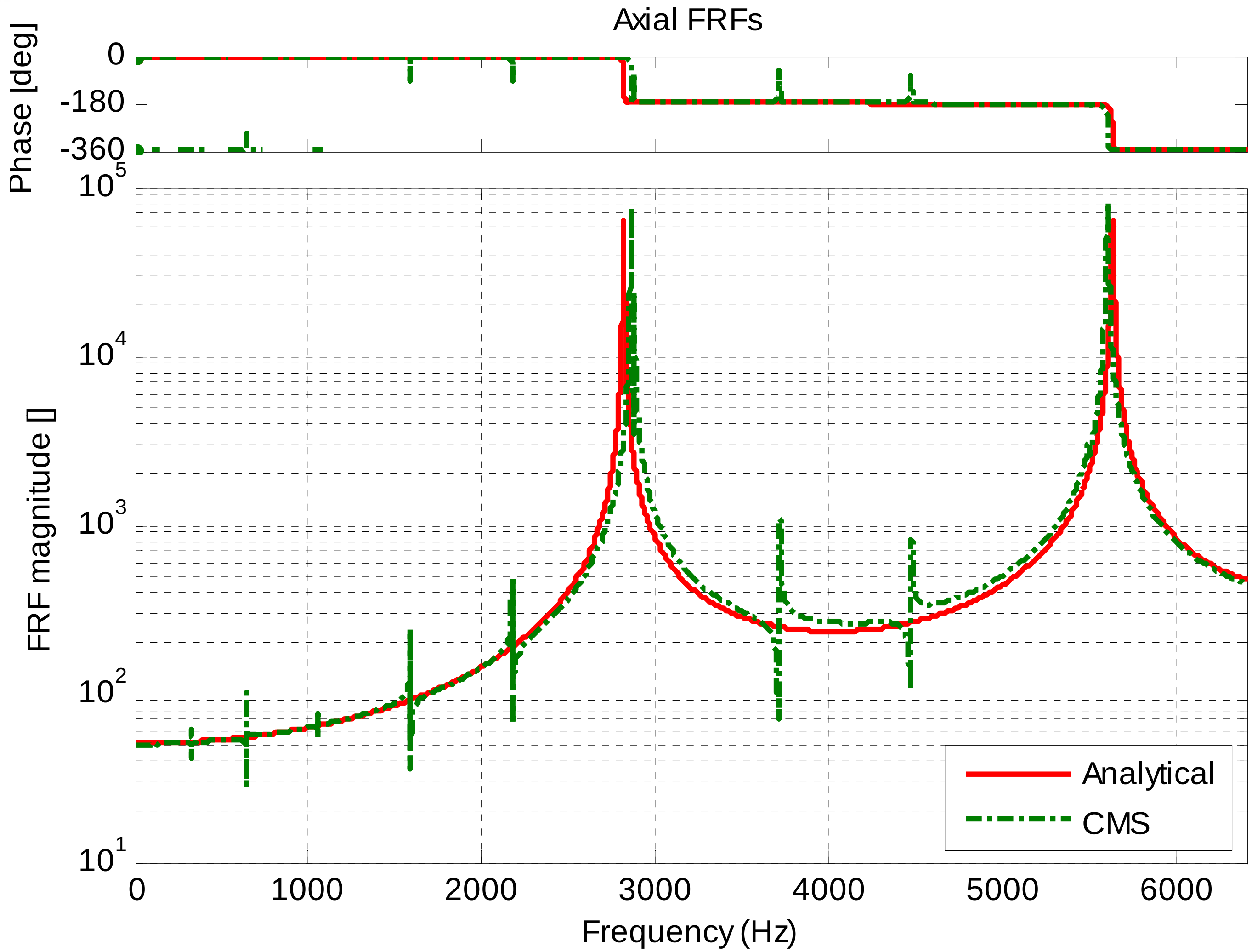
Case 2: Flexible A, MCFS Method



- The CMS method predicts the axial FRF very accurately when the Modal Constraint for Fixture and Subsystem (MCFS) is used.

Physically Realizable Models

- The E system models obtained were not completely physical.
  - Some of the E system models had complex natural frequencies
  - The E system mass matrix was not always positive definite.
- This is to be expected since an approximation to the stiffness of the fixture has been removed.
- When combining structures, the spurious modes all appear at high frequencies, yet they can appear at lower frequencies when removing a substructure.
  - In these cases, the spurious natural frequencies are always near the extremes of the frequency band.
  - Admittance results also show non-physicality (negative eigenvalues of drive point FRFs at some frequencies).

	Complex f_n (Hz)	**eig(M) < 0**
Case 1a **Rigid Fixture, Con. Pt.**	0 + i*8.93e-5 0 + i*54100	-0.011
Case 1b **Flexible Fixture, Con. Pt.**	0 + i*1.36e-4 8951 – i*2450 8951 + i*2450	-1
Case 2 **Flexible Fixture, MCFS**	0 + i*2.28e-4 13050 – i*4285 13050 + i*4285	-0.086

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It could be worse!

	Complex f_n (Hz)	eig(M) < 0
Case 1a **Rigid Fixture, Con. Pt.**	0 + i*8.93e-5 0 + i*54100	-0.011
Case 1b **Flexible Fixture, Con. Pt.**	0 + i*1.36e-4 8951 – i*2450 8951 + i*2450	-1
Case 2 **Flexible Fixture, MCFS**	0 + i*2.28e-4 13050 – i*4285 13050 + i*4285	-0.086

Conclusions

- Lateral (bending direction)
  - Both Component Mode Synthesis (CMS) and FRF based Substructuring (Admittance) can be used to accurately predict the lateral modes of this Experimental-Analytical system.
- Axial direction:
  - It was necessary to account for the elasticity of the fixture to obtain accurate estimates of the axial modes.
  - When doing so, the Modal Constraint for Fixture and Substructure (MCFS) method improved the accuracy of the predictions when compared to the connection point method (CPT).
  - MCFS allows one a new level of freedom when designing test fixtures for these types of analyses.
  - It would be helpful to have a method of removing test fixtures that assures that a physically meaningful model is obtained.

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