

# Continuous-Scan Laser Vibrometry with Unmeasured Input Forces and Application to a 20kW Wind Turbine Shifei Yang & M. S. Allen, University of Wisconsin-Madison, Dept. of Engineering Physics

Grant Title: CMMI-0969224 - Method for Experimental Identification of Nonlinear Dynamic Systems of Unknown Form and Order With Application to Human Gait, Program: Dynamical Systems (DS). Program Manager: Eduardo A. Misawa, PI: Matthew S. Allen, 2011 NSF CMMI Conference, Atlanta, GA, January 2011

#### 1. Motivation:







- Wind Turbines must be carefully designed to avoid failure due to vibration.
- Tests used to verify that the turbine meets design requirements and to characterize wear or damage, but are difficult to perform.
- Difficult to attach sensors to the structure.
- Laser vibrometer alternative:
- Non-contact laser simplifies setup
- Measurements can be performed from the
- However, mode shapes difficult to obtain since test time at each measurement point is long and measurement at several points simultaneously is prohibitively expensive.

# 2. Background:

- Continuous-Scan Laser Vibrometry (CSLDV):
  - Velocity is measured as the laser spot sweeps continuously over the structure (as opposed to measuring each point sequentially).
- Accelerates measurements by effectively measuring the response at many points simultaneously.
- However, existing methods for treating CSLDV measurements require controlled input forces - need a method that can utilize the unmeasured forces exerted by the wind.

## 3. Methods:

Assume Linear Time Invariant Structure (LTI)

$$\dot{x} = \mathbf{A}x + \mathbf{B}u$$
$$\mathbf{v} = \mathbf{C}x + \mathbf{D}u$$

$$M\ddot{q} + C\dot{q} + Kq = F(t)$$

CSLDV: measurement point continuously scans over the structure

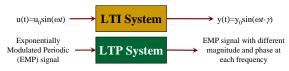
$$\dot{x} = \mathbf{A}x + \mathbf{B}u$$

$$v = \mathbf{C}(t)x$$

Limit to closed (periodic) scanning pattern

$$\mathbf{C}(t) = C(T_A + t)$$

- CSLDV measurements can be treated as the output of a Linear Time Periodic (LTP) system.
- Allen et al. recently created a new frequency domain algorithm for LTP system Identification based on the Harmonic Transfer Function concept by Wereley & Hall:



As for LTI systems, the HTF can be expressed in terms of the modes:

As for ETT systems, the FTF can be expressed in terms of the modes: 
$$\mathbf{G}(\omega) = \sum_{r=1}^{N} \sum_{l=-\infty}^{\infty} \frac{\overline{\mathbf{C}}_{r,l} \overline{\mathbf{B}}_{r,l}}{i\omega - (\lambda_r - il\omega_A)} + \mathbf{D} \qquad \qquad C(t) \psi_r(t) = \sum_{n=-\infty}^{\infty} C_{r,n} e^{jn\omega_A t}$$

$$\overline{\mathbf{C}}_{r,l} = \begin{bmatrix} \cdots & C_{r,-l-l} & C_{r,-l-l} & C_{r,-l-l} & \cdots \end{bmatrix}^T$$

$$\overline{\mathbf{B}}_{r,l} = \begin{bmatrix} \cdots & B_{r,l+1} & B_{r,l} & B_{r,l-1} & \cdots \end{bmatrix}$$
Output Only Identification
$$C(t) \psi_r(t) = \sum_{n=-\infty}^{\infty} C_{r,n} e^{jn\omega_A t}$$

$$L_r(t)^T B(t) = \sum_{n=-\infty}^{\infty} B_{r,n} e^{jn\omega_A t}$$

$$V_r(t) = \sum_{n=-\infty}^{\infty} C_{r,n} e^{jn\omega_A t}$$

Output Only Identification

$$\left[ S_{yy}(\omega) \right] = \mathbf{G}(\omega) S_{uu}(\omega) \mathbf{G}(\omega)^{H} = \sum_{r=1}^{N} \sum_{l=-\infty}^{\infty} \frac{\overline{\mathbf{C}}_{r,l} \mathbf{W}(\omega)_{r,l} \overline{\mathbf{C}}_{r,l}^{H}}{[i\omega - (\lambda_{r} - il\omega_{s})]^{[i\omega - (\lambda_{r} - il\omega_{s})]^{H}}}$$

- - Each mode appears at many frequencies, spaced by integer multiples of fundamental frequency ω<sub>λ</sub>.
  - $f \square$  Mode Shapes  $f \overline{C}_{r,l}$  comprised of coefficients of a Fourier series expansion of the time-varying shapes.

# 4. Experimental Validation:



## 5. Results:

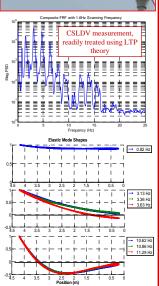
- Conventional OMA
- Laser positioned at a fixed
- □ 10 minutes required to obtain the spectrum.
- Several modes below 15Hz are significantly excited.
- Mode shapes are not obtained.

- Measurement
- point at the tip of the blade.



Mode	Fixed point OMA on tower	on tower
1st Tower	0.81Hz	0.78Hz
Flap Wise Bending 1	3.13Hz	3.13Hz
	3.37Hz	3.36Hz
	3.63Hz	3.62Hz
Edge Wise Bending 1	4.38Hz	-
?	9.13Hz	-
Flap Wise Bending 2	10.63Hz	10.62Hz
	10.94Hz	10.86Hz
	11 25Hz	11 29Hz

- CSI DV Measurement
- Laser scans a line along the length of the blade while measuring.
- Spectrum and mode shapes at many points obtained from two measurements totaling 20 minutes.



# 6. Conclusions & Outlook:

- Output-only CSLDV is experimentally viable and can provide significant time savings for large structures.
- Application to a real turbine blade acquired mode shapes in minutes
- Conventional LDV would require additional equipment and several hours of measurements to obtain the mode shapes.
- Lessons learned are being used to extend LTP output only identification methods to characterize nonlinear systems about periodic limit cycles.

#### References

- A. B. Stanbridge, M. Martarelli, and D. J. Ewins, "Measuring area vibration mode shapes with a continuous-scan LDV," Measurement, vol. 35, pp. 181-9, 2004.
- weasurement, vol. 35, pp. 161-9, 2004.
  M. S. Allen, M. Sracie, S. Chauhan, and M. H. Hansen, "Output-Only Modal Analysis of Linear Time Periodic Systems with Application to Wind Turbine Simulation Data," in 28th International Modal Analysis Conference (IMAC XXVIII) Jackscnville, Florida, 2010.
- Jacksonville, Florida, 2010.

  N. M. Wereley, "Analysis and Control of Linear Periodically Time Varying Systems," PhD Thesis, Department of Aeronautics and Astronautics, Cambridge: Massachusetts Institute of Technology, 1991.