MODELING THE NONLINEAR DAMPING OF JOINTED STRUCTURES USING MODAL MODELS

by

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Abstract

Structures with mechanical joints are difficult to accurately model; even when the natural frequencies of the system remain essentially constant, the damping introduced by the joints is often observed to depend nonlinearly on amplitude. Although models for individual joints have been employed with some success, the modeling of a structure with many joints remains a significant obstacle. This thesis explores applying nonlinear damping models in a modal framework. Instead of modeling each discrete joint within a structure, a nonlinear damping model is used for each mode of interest. This approach assumes that the mode shapes of the structure do not change significantly with amplitude and that there is negligible coupling between modes. The nonlinear Iwan model has had success in modeling the nonlinear damping of individual joints and is used as a modal damping model in this work.

The methodology is first evaluated using numerical simulations of a spring mass system and a finite element model. Initially, simulated experimental data is found by modeling the nonlinear damping of the bolted joints with discrete Iwan models. A modal Iwan model is then fit to simulated measurements and the accuracy of the modal model is assessed. The proposed approach seems to capture the response of the system quite well in both cases, especially at low force levels when macro-slip does not occur.

Measurements are then presented from a two-beam structure with four bolted interfaces in order to characterize the nonlinear damping introduced by the joints. The measurements reveal that each underlying mode of the structure is well approximated by a single degree-of-freedom system with a nonlinear mechanical joint. At low force levels the measurements show dissipation that scales as the second power of the applied force, agreeing with theory for a linear viscously damped system. This is attributed to linear viscous
behavior of the material and/or damping provided by the support structure, which simulates free-free boundary conditions. At larger force levels the damping is observed to behave nonlinearly, suggesting that damping from the mechanical joints is dominant. A model is introduced that captures these effects, consisting of a spring and viscous damping element in parallel with a 4-Parameter Iwan model. The parameters of this model are identified for each mode of the structure and comparisons suggest that the model captures the linear and nonlinear damping accurately over a range of forcing levels.
Nomenclature

Arranged by section and order of appearance

3

$\chi$ - displacement of Jenkins element
$\phi$ - strength of Coulomb frictional slider
$k$ - elastic spring stiffness of Jenkins element
$F$ - force applied to Jenkins element

3.1

$F_S$ - joint slip force Iwan parameter
$K_T$ - joint stiffness Iwan parameter
$\chi$ - power law energy dissipation Iwan parameter
$\beta$ - level of energy dissipation and curve of energy dissipation Iwan parameter
$F'$ - force in the joint
$\rho(\phi)$ - population density of Jenkins elements
$R$ - level of energy dissipation Iwan parameter
$\phi_{\text{max}}$ - displacement at macro-slip
$S$ - accounts discontinuous slope of the force displacement plot at macro-slip
$U'$ - displacement in the joint
$M$ - linear mass matrix
$K_\infty$ - linear stiffness matrix
$F^X$ - vector of external forces
$F^J$ - vector of nonlinear joint forces

4.1

$\hat{F}_S$ - joint slip force modal Iwan parameter
$\hat{K}_T$ - joint stiffness modal Iwan parameter
$\hat{\chi}$ - power law energy dissipation modal Iwan parameter
$\hat{\beta}$ - level of modal energy dissipation modal Iwan parameter
$q_r$ - modal amplitude of the $r^{th}$ mode
$\dot{q}_r$ - modal acceleration of the $r^{th}$ mode
$\omega_{\infty,r}$ - natural frequency without joint stiffness of the $r^{th}$ mode
$\Phi$ - mass normalized mode shape matrix
$\hat{F}_r$ - modal joint force
$K_0$ - stiffness matrix with linear stiffness and joint stiffness
$K_T$ - joint stiffness matrix
$\omega_{0,r}$ - natural frequency with joint stiffness of the $r^{th}$ mode
$\hat{K}_{0,r}$ - modal stiffness with joint stiffness of the $r^{th}$ mode
$\hat{K}_{\infty,r}$ - modal stiffness without joint stiffness of the $r^{th}$ mode

4.2

$\hat{C}$ - modal viscous damper has a coefficient

4.3

$q_0$ - modal amplitude of displacement
$\omega$ - response frequency
$\hat{F}_{LE}$ - force in the linear elastic spring
$\hat{F}_{Iwan}$ - force in the Iwan model
$\hat{R}$ - level of energy dissipation modal Iwan parameter
$\hat{\phi}_{\text{max}}$ - modal displacement at macro-slip
$\hat{F}_{VD}$ - force in the viscous damper
$\hat{F}_{\text{Total1}}$ - total force in the modal Iwan model
$\hat{F}_{\text{Total2}}$ - total force in the modal Iwan model with a viscous damper
\( \dot{D}_{\text{Model}} \) - modal energy dissipation per cycle
\( \dot{D}_{\text{Micro1}} \) - approximate modal energy dissipation per cycle in the micro-slip region for the modal Iwan model
\( \dot{D}_{\text{Micro2}} \) - approximate modal energy dissipation per cycle in the micro-slip region for the modal Iwan model with a viscous damper
\( \dot{D}_{\text{Macro1}} \) - modal energy dissipation per cycle in the macro-slip region for the modal Iwan model
\( \dot{D}_{\text{Macro2}} \) - modal energy dissipation per cycle in the macro-slip region for the modal Iwan model with a viscous damper
\( \hat{K}_{\text{Micro}} \) - approximate modal stiffness of the Iwan joint in the micro-slip region
\( \hat{K}_{\text{Macro}} \) - modal stiffness of the Iwan joint in macro-slip
\( \hat{K}_{\text{Model}} \) - modal stiffness of the Iwan joint
\( \hat{f}_{\text{Model}} \) - natural frequency of the analytical modal Iwan model

5.1

\( \dot{V}(t) \) - analytic representation of modal velocity
\( \tilde{v}(t) \) - Hilbert transform of the modal velocity
\( v(t) \) - modal velocity
\( e^{\phi(t)} \) - decay envelope
\( \omega_d(t) \) - time varying natural frequency
\( \zeta(t) \) - time varying damping ratio
\( \phi \) - phase angle
\( KE \) - modal kinetic energy
\( \dot{D}_{\text{Exp}} \) - modal experimental energy dissipation
\( \tilde{f}_{\text{Exp}} \) - experimental natural frequency

6.2

\( f_D \) - modal energy dissipation objective function
\( f_f \) - natural frequency objective function
\( f \) - objective function
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1. Introduction

Mechanical joints are known to be a major source of damping in assembled structures. However, the physics at a joint interface are quite complex and the amplitude dependence of damping has proven quite difficult to predict. For many systems, linear viscous damping models seem to capture the response of a structure near a single calibrated force level. However, that approach can be over conservative or even erroneous since a linear model does not capture the amplitude dependence of the damping. Thus, it would be far better to understand how mechanical joints behave over a range of forces so that the response of an assembled structure can be modeled accurately.

Friction is known to be the main cause of energy dissipation in jointed structures. The discontinuous nature of friction has led to the definition of two different slip regions, micro-slip and macro-slip. Mechanical joints are said to be undergoing micro-slip when the stiffness of the joint remains intact but small slip displacements occur at the outskirts of the contact patch causing frictional energy loss in the system [1]. Macro-slip occurs when the stiffness of the joint is compromised and relatively large slip displacements are possible.

To capture the nonlinear damping associated with jointed structures, a plethora of models have been developed based on the Coulomb friction model, some of which are reviewed in [2-4]. A popular element used to model friction contains a spring in series with a Coulomb frictional damper, and is often referred to as a Jenkins element in previous literature (also macroslip element, bilinear hysteresis element). Iwan used a distribution of Jenkins elements in parallel to model the elasto-plastic hysteretic behavior of materials and structures [5]. Several distributions of Jenkins elements have been used since then to model
joints [6-10]. A four parameter distribution function is used in this thesis, known as the 4-Parameter Iwan model [11]. This model accounts for the key characteristics of a joint's response including the joint slip force \( F_s \), joint stiffness \( K_i \), and power law energy dissipation \( \chi, \beta \). In the past decade, the 4-Parameter Iwan model has been implemented to predict the vibration of structures with a few discrete joints [12, 13]. However, when modeling individual joints, each joint requires a unique set of parameters, which means that one must deduce hundreds or even thousands of joint parameters to describe a system with a large number of joints. On the other hand, when a small number of modes are active in a response, recent measurements have suggested that a simpler model may be adequate. Segalman recently applied the 4-Parameter Iwan model in a modal framework to describe two simple spring mass systems [14]. However, little work has been done to validate the modal Iwan approach using real structures.

This thesis explores the validity of the modal Iwan approach applied to a realistic finite element model and experimental measurements of a bolted structure. The modal Iwan model is shown to capture the nonlinear damping of bolted structures quite well. In addition, the modal Iwan framework is extended by adding some features that are necessary to approximate real experimental data. The 4-Parameter Iwan model was developed to model the energy dissipation associated with joints. However, this model may not be the best representation for other sources of experimental energy dissipation which include: the support conditions of the experimental setup, and the light material damping of the test specimen. To account for these sources of damping, a viscous damper is used in parallel with a modal Iwan model to fit experimental measurements.
The results of the thesis are organized as follows. First, an academic model or spring mass system is used to simulate measurements of a discrete Iwan joint, from which modal Iwan parameters are deduced. Then, a more realistic finite element structure, with four locations of discrete Iwan joints, is modeled using finite elements and once again the simulated response data is used to deduce a set of modal Iwan parameters. Particular attention is given to the degree to which the modal Iwan model captures the response of the system with discrete Iwan joints. Finally, the methodology is applied experimentally to a bolted structure. In order to assure that these deduced modal models accurately model the system, they are validated by comparing time responses and frequency spectra.

2. Slip Regions

Nonlinearities associated with mechanical joints are often classified into two different regions, micro-slip and macro-slip. Consider the joint shown in Figure 1, where a bolt is used to connect two slabs of material. The preload in the bolt creates a contact region between the two slabs near the bolt. If a force F is applied to the slabs, slip will occur at the outskirts of the contact region. This means that there will be a region of stick and a region of slip as indicated in Figure 1. For this case, the bolted joint is said to be undergoing micro-slip due to the small slip displacements that cause the frictional energy loss [1].
Figure 1: Contact and Slip regions are shown for a mechanical joint undergoing micro-slip [1].

Macro-slip occurs when the stick region vanishes and larger slip displacements are possible. If a very large displacement is considered, the slabs of material will eventually come into contact with the bolt. This further complicates the system and is not considered in this thesis.

3. Discrete Joint Models

To capture the nonlinear damping of joints in both the micro-slip and the macro-slip regions, a Jenkins element, based on the Coulomb friction model, is often used. A Jenkins element consists of a spring in series with a Coulomb frictional damper as shown in Figure 2.

Figure 2: Schematic of a single Jenkins element.
The Jenkins element from Figure 2 has a strength, $\phi$ in units of length [11], and a stiffness, $k$. In order to accurately model an actual joint, several Jenkins elements are often placed in parallel with one another. A parallel arrangement of Jenkins elements, as seen in Figure 3, allows for some of the elements to slip, while others remain stuck. Therefore, multiple Jenkins elements seem to describe the micro-slip phenomena, where frictional energy loss is apparent yet the stiffness of the joint remains intact.

![Figure 3: Schematic of a parallel arrangement of Jenkins elements [11].](image)

Originally, Iwan used a distribution of Jenkins elements in parallel to model the elasto-plastic hysteretic behavior of materials and structures [5]. A historical review of the major contributors prior to Iwan's work led to this model being referred to as the Bauschinger-Prandtl-Ishlinskii-Iwan (BPII) type model in a more recent work [7]. However, for simplicity and consistency with literature, in this thesis the model will be referred to as the Iwan model.
Since Iwan's work, several distributions or population densities of Jenkins elements have been used to model joints [6-10]. A population density with four parameters is used in this thesis and is known as the 4-Parameter Iwan model [11]. It should be mentioned that the group at the University of Stuttgart, lead by Gaul et al., have modeled joints using a single modified Jenkins element with the signum function smoothed by an exponential function [3]. This approach was not explored in this thesis, but gives an example of a similar method that has been used to model joints.

3.1. 4-Parameter Iwan Model

The 4-Parameter Iwan model, initially presented in [11], accounts for the key characteristics of a joint's response including the joint slip force \(F_S\), joint stiffness \(K_T\), and power law energy dissipation \((\chi, \beta)\). The force in the joint, \(F_J(t)\), is shown to have the following form,

\[
F_J(t) = \int_0^\infty \rho(\phi)[u(t) - x(t, \phi)] d\phi
\]  

where

\[
\dot{x}(t, \phi) = \begin{cases} 
\dot{u} & \text{if } \|u - x(t, \phi)\| = \phi \text{ and } \dot{u}[u - x(t, \phi)] > 0 \\
0 & \text{otherwise}
\end{cases}
\]

and \(u(t)\) is the extension of the joint, \(x(t, \phi)\) is the displacement of the frictional damper with strength \(\phi\) and \(\rho(\phi)\) is the population density of the Jenkins elements with strength \(\phi\). To accommodate the behavior of joints at a large range of forces, a 4-parameter population density was developed with the form [11]:
\[
\rho(\phi) = R\phi^\alpha \left[ H(\phi) - H(\phi - \phi_{\text{max}}) \right] + S\delta(\phi - \phi_{\text{max}})
\]
\[(3)\]

where \(H(\phi)\) is the Heaviside step function, \(\delta(\phi)\) is the Dirac delta function and the four parameters that characterize the joint include: \(R\), which is associated with the level of energy dissipation, \(\chi\), which is directly related to the power law behavior of energy dissipation, \(\phi_{\text{max}}\), which is equal to the displacement at macro-slip, and the coefficient \(S\), which accounts for a potential discontinuous slope of the force displacement plot when macro-slip occurs. The population density, given in Eq. (3), is shown in the graphical form in Figure 4.

![Population Density](image-url)

**Figure 4:** Population density of a the 4-Parameter Iwan model [11].

The four parameters \(\{R, S, \chi, \phi_{\text{max}}\}\) are converted to more physically meaningful parameters \(\{F_s, K_T, \chi, \beta\}\) in [11, 13]. \(F_s\) is the joint force necessary to initiate macro-slip, \(K_T\) is the stiffness of the joint, \(\chi\) is directly related to the slope of the energy dissipation in the micro-slip regime, and \(\beta\) relates to the level of energy dissipation and the shape of the energy dissipation curve as the macro-slip force is approached. This new set of parameters, \(\{F_s, K_T, \chi, \beta\}\) can be deduced from the two plots seen in Figure 4.
Figure 5: (a) Both macro-slip force \( F_S \) and joint stiffness \( K_T \) can be found from the force-displacement relationship of the joint. (b) The \( \chi \) and \( \beta \) values are found from the dissipation curve.

Figure 4a plots the joint force, \( F^J \), versus the joint displacement, \( U^J \) and Figure 4b plots the logarithm of the joint force versus the logarithm of the dissipation/cycle in the joint. Given a set of simulation or experimental data, these plots can be used to identify discrete Iwan parameters.

If we consider a system where the damping is strictly due to the 4-Parameter Iwan model, the governing equation becomes

\[
M\ddot{x} + K_\infty x = F^X + F^J
\]

where \( M \) and \( K_\infty \) are the linear mass and stiffness matrices of a finite element model, \( F^X \) is the vector of external forces, and \( F^J \) is a vector of nonlinear joint forces that the joint applied to the structure. Note that \( F^J \) is the 4-parameter Iwan model defined in Eq. (1) with opposite sign convention and that \( K_\infty \) does not contain the stiffness associated with the joint, \( K_T \).
has nonzero entries corresponding to the two ends where the joint is attached to the finite
element model and depends on the displacement at those points.

As mentioned previously, the 4-parameter Iwan model has been implemented to
predict the response of structures with a small number of discrete joints [12, 13]. However,
each joint requires a unique set of parameters \(\{F_s, K_s, X, \beta\}\), so many joint parameters need to
be deduced and it is not likely that they can all be determined uniquely from a set of
experimental measurements.

4. Modal Joint Models

In order to circumvent this limitation, Segalman proposed that energy dissipation be
applied on a mode-by-mode basis, using the 4-parameter Iwan model [14]. In general, the
nonlinearity that joints introduce can couple the modes of a system so that modes in the
traditional linear sense can not be defined. However, if the structure is lightly damped, the
nonlinear effect is relatively weak and experiments have shown that the mode shapes
typically do not change significantly, suggesting that one might be able to model the
structure as a collection of uncoupled linear modes, each with nonlinear damping
characteristics [15]. The Iwan model is assumed to be applicable to each mode, although the
parameters of each mode are tuned to match experimental measurements and hence aren't
necessarily the same as the parameters of any individual joint in the structure. This allows
one to model a structure with a relatively small number of modes that capture its
performance in the frequency band of interest, and the response of each mode is found
through a nonlinear, single degree-of-freedom simulation.
4.1. Modal Iwan Model

Under these assumptions, each modal degree-of-freedom is modeled by a single degree-of-freedom oscillator, as shown in Figure 6, with a 4-parameter Iwan model placed between the modal degree of freedom and the ground. A second spring is placed in parallel with the 4-parameter Iwan model, representing the residual stiffness of that mode when the Iwan model is in macro-slip. Note that the displacement of the mass is not a physical displacement but the modal displacement or modal amplitude, \( q \), of the mode of interest. The mode vectors are assumed mass normalized so the mass is taken to be unity.

\[
\begin{align*}
\hat{F}_s, \hat{K}_T, \hat{\chi}, \hat{\beta} \quad &\quad \hat{K}_\infty \\
M = 1 \quad &\quad q
\end{align*}
\]

**Figure 6:** Schematic of the modal Iwan model for each modal degree-of-freedom. Each mode has a unique set of parameters that characterize its nonlinear damping.

The 4-parameter modal Iwan model has parameters \( \{\hat{F}_s, \hat{K}_T, \hat{\chi}, \hat{\beta}\} \) where \( \hat{F}_s \) is the modal joint force necessary to initiate macro-slip, \( \hat{K}_T \) is the modal stiffness of the joint, \( \hat{\chi} \) is directly related to the slope of the modal energy dissipation in the micro-slip regime, and \( \hat{\beta} \) relates to the level of the modal energy dissipation and the shape of this curve as the macro-slip force is approached, and the linear elastic spring stiffness is \( \hat{K}_\infty \). Again, the modal Iwan
parameters, denoted with a hat, \( \{ \hat{F}_s, \hat{K}_r, \hat{X}, \hat{\beta} \} \) will in general not be the same as any of the discrete Iwan parameters \( \{ F_s, K_r, \chi, \beta \} \). The response for each mode \( r \) is then governed by the following differential equation:

\[
\ddot{q}_r + \omega_{0,r}^2 q_r = \Phi_r^T \Phi X + \hat{F}_r
\]

where \( \hat{F}_r \) is the modal joint force that takes the same form as the discrete joint force defined in Eq. (1), except instead of the discrete Iwan parameters, the modal Iwan parameters are used inserted into Eq. (1). The low force mode shapes are used to define the modal parameters. Therefore, the stiffness matrix used in the Eigen value problem must include the low-force stiffness of the joints. The mode shape vectors, \( \Phi_r \), are found by solving the Eigen value problem:

\[
[ K_0 - \omega_{0,r}^2 ] \Phi_r = 0
\]

\[
K_0 = K_\infty + K_r
\]

where \( K_r \) is a matrix that captures the stiffness that the joints contribute to the structure. Note that the modal stiffness parameters and the frequencies are related by the following equations:

\[
\omega_{0,r} = \sqrt{\hat{K}_{0,r}}
\]

\[
\omega_{\infty,r} = \sqrt{\hat{K}_{\infty,r}}
\]

In practice, these modal parameters will come from a low-level modal test and so this assumption should be valid. On the other hand, if the structure is modeled in finite elements
then the stiffness of the joints should be included when calculating the mode shapes; this functionality is built into the Sierra/SD (Salinas) finite element package [16, 17].

4.2. Modal Iwan Model with a Viscous Damper

The 4-Parameter Iwan model was developed to model the energy dissipation associated with joints. However, this model may not be the best representation for other sources of experimental energy dissipation which include: the support conditions of the experimental setup, and the light material damping of the test specimen. To account for these sources of damping, a viscous damper was added in parallel with a 4-Parameter Iwan modal model to fit experimental measurements. With this additional consideration, each modal degree-of-freedom is modeled by a single degree-of-freedom oscillator, as shown in Figure 7, with a 4-parameter Iwan model in parallel with a viscous damper and an elastic spring.

![Figure 7: Schematic of the model for each modal degree of freedom with an Iwan model and viscous damper.](image)
The modal Iwan model with a viscous damper is identical to the model in Figure 6, with the addition of a viscous damper that has a coefficient, $\hat{C}$. Note again that all of the parameters are defined in modal and not physical space.

4.3. Analytical Models for Energy Dissipation and Frequency

The energy dissipation and frequency content of the modal joint models can be solved for and used to fit experimental data. Consider either of the joint models presented in Figure 6 or Figure 7, assuming a harmonic load is applied to the mass and the system is at steady-state, the mass will oscillate as

$$q = q_0 \sin(\omega t)$$  \hspace{1cm} (10)

where $q_0$ is the modal displacement amplitude, and $\omega$ is the response frequency. The force in the linear elastic spring for both joint models takes the form

$$\hat{F}_{LE} = \hat{K}_\infty q$$  \hspace{1cm} (11)

where $\hat{K}_\infty$ is the spring stiffness. The force in the Iwan model is given in [11]. Assuming that the amplitude of motion is small, $q_0 < \phi_{\text{max}}$ or in other words the joint is undergoing micro-slip, the force in the Iwan model can be approximated as

$$\hat{F}_{Iwan} = \hat{R}q^{\hat{z}+2}$$  \hspace{1cm} (12)

where $\hat{R}$ is a coefficient that describes the population distribution of the parallel-series Iwan system [11]. As mentioned for the discrete Iwan models, the modal Iwan parameter $\hat{R}$ can be written in terms of physically meaningful parameters:
\[
\hat{R} = \frac{\hat{F}_S (\hat{\chi} + 1)}{\hat{\phi}_{max} (\hat{\beta} + \hat{\chi} + 1)}
\]

where

\[
\hat{\phi}_{max} = \frac{\hat{F}_S (1 + \hat{\beta})}{\hat{K}_T (\hat{\beta} + \hat{\chi} + 1)}
\]

Finally, if we consider the modal joint model with the Iwan model in parallel with a viscous damper, the force in the viscous damper can be written as

\[
\hat{F}_{VD} = \hat{C} \dot{q}
\]

where \(\hat{C}\) is the modal viscous damping coefficient. These forces can be added, for each modal joint model so that \(\hat{F}_{Total1} = \hat{F}_{LE} + \hat{F}_{Iwan}\) for the model in Figure 6 and \(\hat{F}_{Total2} = \hat{F}_{VD} + \hat{F}_{LE} + \hat{F}_{Iwan}\) for the model in Figure 7. The total forces are then multiplied by the modal velocity and integrated over one period as follows,

\[
\hat{D}_{Model} = \int_0^{2\pi/\omega} \hat{F}_{Total} \dot{q} dt
\]

to obtain the energy dissipation per cycle. The energy dissipation in the micro-slip regime for the modal Iwan model alone is \(\hat{D}_{Micro1}\) and with the viscous damper \(\hat{D}_{Micro2}\).

\[
\hat{D}_{Micro1} \approx \frac{4\hat{R}q_0 \hat{\chi}^3}{(\hat{\chi} + 3)(\hat{\chi} + 2)}
\]

\[
\hat{D}_{Micro2} \approx \frac{4\hat{R}q_0 \hat{\chi}^3}{(\hat{\chi} + 3)(\hat{\chi} + 2)} + \pi \omega \hat{C} \dot{q}_0^2
\]
Notice that the energy dissipation depends on the maximum modal amplitude \( q_0 \) and that the linear elastic spring does not contribute to the energy dissipated as one would expect.

In the macro-slip region, the force in the Iwan joint has become \( \hat{F}_{Iwan} = \hat{F}_S \).

Therefore, the modal energy dissipation is given by:

\[
\hat{D}_{Macro 1} = 4q_0 \hat{F}_S \\
\hat{D}_{Macro 2} = 4q_0 \hat{F}_S + \pi \omega \hat{C} q_0^2
\]

for the two models from Sec. 4.1 and 4.2 respectively. Therefore, the total energy dissipation can be written for either model as:

\[
\hat{D}_{Model} = \begin{cases} 
\hat{D}_{Micro} & \text{if } \hat{F}_J < \hat{F}_S \\
\hat{D}_{Macro} & \text{if } \hat{F}_J \geq \hat{F}_S 
\end{cases}
\]

For both models, the secant stiffness of the Iwan joint in the micro-slip region can be approximated as [11]:

\[
\hat{K}_{Micro} \approx \hat{K}_T \left( 1 - \frac{\hat{\dot{\chi} + 1}}{(\hat{\dot{\chi}} + 2)(\hat{\beta} + 1)} \right) + \hat{K}_e
\]

where

\[
\hat{F} = \frac{q_0 \hat{K}_T \left( \hat{\beta} + \frac{\hat{\dot{\chi}} + 1}{\hat{\dot{\chi}} + 2} \right)}{\hat{F}_S (1 + \hat{\beta})}
\]

In the macro-slip region, the stiffness is given by:

\[
\hat{K}_{Macro} = \hat{K}_e
\]
Therefore, the total energy dissipation can be written as:

\[
\hat{K}_{\text{Model}} = \begin{cases} 
\hat{K}_{\text{Micro}} & \text{if } \hat{F}^s < \hat{F}_S \\
\hat{K}_{\text{Macro}} & \text{if } \hat{F}^s \geq \hat{F}_S 
\end{cases}
\]  

(25)

assuming mass normalized mode shapes are used, the natural frequency of the analytical model is

\[
\hat{f}_{\text{Model}} = \frac{1}{2\pi} \sqrt{\hat{K}_{\text{Model}}}
\]

(26)

Note that both energy dissipation and frequency in the micro-slip regions are approximations to the actual dissipation and stiffness. In order to obtain the actual dissipation and frequency, the Iwan model can be integrated in time and then the actual dissipation and frequency can be deduced. However, as discussed in later sections, \( \hat{D}_{\text{Model}} \) and \( \hat{f}_{\text{Model}} \) are used to decrease computational time when solving an optimization problem to find the modal Iwan parameters that best fit the data.

5. Transient Excitation Analysis

In this thesis, the modal parameters for the models discussed above will be extracted from both simulations and experimental measurements where a transient excitation was applied to the structure. A modal response ring-down can be used to estimate the frequency and energy dissipation for each mode of interest. Various methods for calculating the frequency and energy dissipation have been used in previous works \([15, 18, 19]\). The procedure for processing measurements will be reviewed below.
5.1. Processing Transient Excitation Measurements

First, a filter is used to isolate an individual modal response. Both modal filters [20] and standard, infinite impulse response band-pass filters [21] have been used for this purpose and other possibilities certainly exist. The Hilbert Transform [22] is then used to compute the instantaneous damping and frequency of the system. This process requires some care since the basic Hilbert transform performs very poorly in the presence of noise. This thesis uses a variant where curve fitting is used [23] to smooth the instantaneous amplitude and phase found by a standard Hilbert transform and then the curve fit model can be differentiated to estimate the energy dissipation and instantaneous frequency, as explained below.

In the following section, velocity measurements are assumed to be taken in an experiment using a laser Doppler vibrometer, which was the technique used in the experimental portion of this thesis. One obtains an analytic representation of the modal response, denoted \( \hat{V}(t) \), by adding the Hilbert transform of the modal velocity, \( \hat{v}(t) \), to the measured modal velocity of the mode of interest, \( v(t) = \dot{q}_r(t) \) as follows

\[
\hat{V}(t) = v(t) + i\hat{v}(t)
\] (27)

The magnitude of the analytic signal is the decay envelope of the response and is approximated by

\[
|\hat{V}(t)| = V_0 e^{P(t)}
\] (28)
where \( V_0 \) is the initial amplitude and \( e^{P(t)} \) is the decay envelope. To maintain similarity with a linear system, the product of the natural frequency, \( \omega_n(t) \), and the coefficient of critical damping, \( \zeta(t) \), is defined to be the time derivative of \( P(t) \).

\[
\frac{dP(t)}{dt} = \alpha(t) = -\zeta(t)\omega_n(t)
\]  
(29)

The instantaneous phase is the complex angle of the analytic signal, which can be obtained using the following (provided that a four-quadrant arctangent formula is used).

\[
\phi(t) = \tan^{-1}\left( \frac{\hat{v}(t)}{v(t)} \right)
\]  
(30)

The measured phase and the natural logarithm of the decay envelope are then smoothed by fitting a polynomial to the data. In addition, before the data is fit, the beginning and end of the data are deleted since they tend to be contaminated by end effects in the Hilbert Transform. The time-derivative of the phase then gives the instantaneous damped natural frequency.

\[
\omega_d(t) = \frac{d\phi(t)}{dt}
\]  
(31)

The time varying natural frequency is then found using the following equation:

\[
\omega_n(t) = \sqrt{(\omega_d(t))^2 + (-\alpha(t))^2}
\]  
(32)

Now the energy dissipation per cycle can be calculated from the change in kinetic energy over one cycle. The amplitude of the kinetic energy can be written as,

\[
KE = \frac{1}{2} M \left| \hat{V}(t) \right|^2
\]  
(33)
and the change in the kinetic energy is found by taking the derivative of this expression. Since the kinetic energy and its derivative are quite smooth, the energy dissipated per cycle, $\hat{D}_{\text{Exp}}$, can be approximated by simply multiplying $dKE/dt$ by the period $(2\pi/\omega_d(t))$ (e.g. using a trapezoid rule to integrate the power dissipated as a function of time). This assumes that the kinetic energy changes slowly relative to the period so that it can be approximated as a constant over each cycle.

$$\hat{D}_{\text{Exp}} \approx \frac{2\pi}{\omega_d} \frac{dKE}{dt} = -\frac{2\pi}{\omega_d} \frac{dP(t)}{dt} \left| \dot{\gamma}(t) \right|^2$$

(34)

Finally, the time varying natural frequency is converted from radians per second to Hertz and is denoted as the modal experimental frequency.

$$\hat{f}_{\text{Exp}} = \frac{\omega_d(t)}{2\pi}$$

(35)

In order to compute the modal joint model's energy dissipation, Eq. (21), a displacement ring-down is needed from the experiment. This was obtained by integrating the measured velocity signal with respect to time using a trapezoidal numerical integration. The same procedure used on the velocity was then used on the displacement. The displacement ring-down was then used to compare the dissipation model (Eq. 21) to the measured experimental data (Eq. 34) in the optimization procedure, as will be discussed.

6. Deducing Modal Parameters

This section explains how modal parameters, including $\{\hat{F}_s, \hat{K}_r, \hat{K}_n, \hat{\gamma}, \hat{\beta}, \hat{C}\}$, are deduced from energy dissipation and frequency versus force measurements. Two approaches will be described in this thesis. The first involves a graphical method of determining the
modal parameters. The second method utilizes an optimization routine to best fit the modal parameters using the graphical method values as a starting guess.

6.1. Graphical Method

The graphical procedure closely follows the work done by Guthrie, which was explained in an unpublished memo. The methods described in the previous section provide a set of frequency and energy dissipation data such as that shown schematically in Figure 8. Note that these are quite similar to those shown in Figure 5 for a discrete joint, except that all of the quantities, such as the slip force, are now in a modal form rather than physical. Also, note that since the modal Iwan model has a linear spring in parallel, its stiffness does not go to zero at macro-slip but instead it simply drops to \( \hat{K}_\infty \).

Figure 8: (a) Modal macro-slip force (\( \hat{F}_s \)) and modal joint stiffness (\( \hat{K}_r \)) can be found from the softening seen in the natural frequency. (b) The \( \hat{\chi} \) value is found from the slope of the modal dissipation and the \( \hat{\beta} \) value is a measure of modal dissipation level and shape of the modal dissipation curve.
The $\hat{\chi}$ parameter is found by fitting a line to the logarithm of energy dissipation versus logarithm of the modal force at low force levels. Then the $\hat{\chi}$ parameter for each mode $r$ is given by the slope of that line minus three:

$$\hat{\chi}_r = \text{Slope}_r - 3 \quad (36)$$

In order to deduce the Iwan modal stiffness $\hat{K}_r$, the natural frequencies of each mode are plotted versus modal joint force. A softening of the system, characterized by a drop in frequency, illustrates the amount of modal stiffness associated with all the relevant joints of the system. Assuming that the mode shapes are mass normalized, the equation for modal joint stiffness for each mode becomes

$$\hat{K}_{T,r} = \hat{K}_{0,r} - \hat{K}_{\infty,r} = \omega_0^2 - \omega_{\infty,r}^2 \quad (37)$$

where $\omega_0$ is the natural frequency corresponding to the case when all the joints in the structure exhibit no slipping, and $\omega_{\infty}$ is the natural frequency when all of the joints are slipping. This points to the major advantage of using a modal Iwan implementation as opposed to a discrete Iwan implementation. For a modal Iwan implementation, only the modal stiffness and modal slip force of each kept mode needs to be considered as opposed to calculating the stiffness and slip force associated with each joint individually. However, when taking experimental measurements, macro-slip is not always clearly observed. When defining a starting guess for the optimization process, the $\omega_{\infty}$ values were assumed to be slightly lower than the lowest measured frequency value.

The modal joint slip force, $\hat{F}_s$, can be estimated from Figure 8a. However, if experimental measurements were taken that did not quite reach the macro-slip levels, the $\hat{F}_s$
value was assumed to be slightly higher than the largest modal force measured. To find the last parameter, \( \hat{\beta} \), all of the previous parameters found are needed along with the y-intercept, \( \hat{A} \), of the line that was fit in order to find the \( \hat{\chi} \) parameter. Then, the following equation was formed from [11] that can be used to solve for \( \hat{\beta} \) numerically.

\[
\hat{F}_{s,r} = \left[ \frac{4(\hat{\chi} + 1)\hat{K}_{T,r}^{\chi,r+2}(\hat{\beta} + \hat{\chi} + 1)}{\hat{A}_{r}\hat{K}_{\chi,r}^{(3+\hat{\chi})}(2 + \hat{\chi})(3 + \hat{\chi})(1 + \hat{\beta})^{\chi,r+2}} \right]^{\frac{1}{\hat{\chi}+1}}
\]

(38)

Once all of these parameters have been determined, one can reconstruct \( \hat{D}_{\text{Model}} \) and \( \hat{f}_{\text{Model}} \) to use in comparison with \( \hat{D}_{\text{Exp}} \) and \( \hat{f}_{\text{Exp}} \).

6.2. Optimization Method

The primary method used in this thesis for deducing the modal joint parameters was to optimize the parameters to best fit the experimental data. The objective function is posed as:

\[
\text{Min } f = f_D + f_f
\]

(39)

where

\[
f_D = \left( \frac{\hat{D}_{\text{Exp}} - \hat{D}_{\text{Model}}}{\max(\hat{D}_{\text{Exp}} - \hat{D}_{\text{Model}})} \right)^2
\]

(40)

and
Note that the dissipation and frequency objective functions, \( f_D \) and \( f_f \) respectively, are weighted so that their values are on the same order of unit magnitude.

The nonlinear objective function, Eq. 39, can be optimized using either local or global optimization. Both techniques were explored by the author; however, when multiple local minima exist, local optimization algorithms tended to be highly dependent on the starting guess. Therefore, a global optimization algorithm (the DIRECT algorithm developed by Jones et al. [24]) was used to provide a more robust approach to optimizing the parameters. In addition, local optimization routines were used in MATLAB (fminsearch, fmincon, lsqnonlin [25]) to fine tune the solution and ensure convergence. Even with the global optimization algorithm, it was important to have a reasonable starting guess. For this work, starting guesses for the \( \{\hat{F}_s, \hat{K}_r, \hat{K}_w, \hat{\epsilon}, \hat{\beta}, \hat{C}\} \) parameters were found using the graphical approach described in the previous section.

7. Numerical Simulations

In the previous sections, the signal processing of experimental results and the optimization of modal model parameters was discussed. In order to demonstrate these proposed techniques, several numerical simulations were run to simulate experimental results. For the experimental simulations, models with discrete Iwan joints were taken as the experimental data. Then, modal model parameters were optimized and compared to the truth model to evaluate the proposed procedure.
7.1. Academic Simulations

Segalman was the first to formulate a modal approach to model the nonlinear damping in joints using academic models [14]. The same three degree-of-freedom academic model was used in this thesis to examine the signal processing and optimization procedures.

7.1.1. Academic Model

![Diagram of the academic model with a discrete Iwan model between the 2nd and 3rd mass](image)

Figure 9: Schematic of the academic model with a discrete Iwan model between the 2nd and 3rd mass [14].

The academic model, as seen in Figure 9, consists of three identical masses connected with linear elastic springs of stiffness, $K_\infty$. A discrete Iwan model is placed between the second and third mass. The parameter values for the academic model are given in Table 1.

Table 1: Parameter values for academic model.

| $M_0$ = 10 | $K_\infty$ = 9 | $K_T$ = 1 | $F_S$ = 10 | $\chi$ = -0.5 | $\beta$ = 5 |

An Eigen analysis is performed with a stiffness matrix that includes the joint stiffness. The mass normalized mode shapes are shown in Table 2.
Table 2: Mass Normalized Mode Shapes for the academic model.

<table>
<thead>
<tr>
<th></th>
<th>1st Mode ($f_1 = 0.0676$ Hz)</th>
<th>2nd Mode ($f_2 = 0.1935$ Hz)</th>
<th>3rd Mode ($f_3 = 0.2776$ Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>-0.11</td>
<td>0.24</td>
<td>-0.17</td>
</tr>
<tr>
<td>$u_2$</td>
<td>-0.19</td>
<td>0.09</td>
<td>0.24</td>
</tr>
<tr>
<td>$u_3$</td>
<td>-0.23</td>
<td>-0.18</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

The mode shapes show that the discrete Iwan joint between the second and third mass will dissipate energy in the 2nd and 3rd modes, since the masses are moving out of phase. However, for the 1st mode all of the masses are moving in phase and the energy loss due to frictional slip in the Iwan model is less severe.

7.1.2. Discrete Iwan (Experimental) Simulations

An experiment was simulated by exciting the model with an impulsive force. The force was applied to third mass as shown in Figure 9. A half period sine wave was used as the forcing function with a duration of approximately 7.40, 2.58, and 1.80 seconds for the first, second, and third modes respectively. The duration of the force was chosen to primarily excite the mode of interest and changes the response level of the masses for each mode. For the second and third modes, six impacts with amplitudes of 5, 10, 20, 60, 500, and 3500 were used to excite the structure in both the micro-slip and macro-slip regions. For the first mode, fifteen impacts with amplitudes of 5, 10, 20, 25, 28, 30, 40, 50, 60, 80, 100, 200, 500, 1000, and 3500 were used.

The equations of motion in physical coordinates were integrated in time with the average acceleration Newmark-Beta integration scheme with an iterative Newton-Raphson
loop to solve the residual force equation. The physical response converted into a modal response using the inverse of the mode shape matrix from an Eigen analysis,

$$ q = \Phi^{-1}x $$

(42)

where $\Phi$ is the mode shape matrix.

The procedure described in Sec. 5.1, or the Hilbert transform with polynomial fitting procedure, was used to find the energy dissipation and frequency of the simulated experimental data. Figure 10 shows the fitting process used on the 1st mode's response after an impact with an amplitude of 500 has been applied to the academic model.

Figure 10: Fitting the 1st mode's modal response and phase using the Hilbert transform with polynomial fitting procedure with a first order polynomial.
The magnitude of the modal velocity and the phase angle are plotted versus time. The first mode is shown to dissipate very little energy. If a high order polynomial is fit to the somewhat flat modal velocity, the fitting process will often produce a decay envelope (Eq. 28) with a positive derivative. Essentially, this means the energy dissipated per cycle is negative, or that energy is added to the system. For this reason, a 1st order polynomial was fit to the first mode's response to ensure that it always decayed over the fit region and the energy dissipation was positive. More impacts were used for the first mode than the other modes since each impact decays very little over a reasonable integration time.

![Academic Model 3rd Mode](image)

**Figure 11:** Fitting the 3rd mode's modal response and phase using the Hilbert transform with polynomial fitting procedure with a 20th order polynomial.
Figure 11 shows the fitting process used for the 3\textsuperscript{rd} mode's response after the same impact is applied to the system. For the 2\textsuperscript{nd} and 3\textsuperscript{rd} modes, a high order polynomial (in this case 20\textsuperscript{th} order) was fit to the data since there is more curvature in the decaying response amplitude. Comparing Figure 10 with Figure 11, notice that the 1\textsuperscript{st} mode oscillates approximately 34 times in 500 seconds while the 3\textsuperscript{rd} mode oscillates approximately 139 times in the same time frame. When computing the energy dissipation per cycle (Eq. 34), it is important to remember that the higher frequency modes contain a greater number of cycles and thus more energy dissipation for a given span of time. Also, note that the amplitude of response for the 1\textsuperscript{st} mode is larger than that of the 3\textsuperscript{rd} mode. Recall that the duration of the applied force was longer for the 1\textsuperscript{st} mode and thus the impact force has inputted more energy into the system and more response was observed.

7.1.3. Modal Iwan Simulations

The methodology described in Sec. 6.2 was used to deduce the modal Iwan parameters from the experimental simulations data of the third mode of vibration. The optimized parameters for the third mode of the academic model are shown in Table 3.

<table>
<thead>
<tr>
<th>Modal Iwan Parameters</th>
<th>1\textsuperscript{st} Mode Optimized Values</th>
<th>2\textsuperscript{nd} Mode Optimized Values</th>
<th>3\textsuperscript{rd} Mode Optimized Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{F}_s )</td>
<td>0.4156</td>
<td>2.8645</td>
<td>3.3893</td>
</tr>
<tr>
<td>( \hat{K}_r )</td>
<td>0.0018956</td>
<td>0.078198</td>
<td>0.11699</td>
</tr>
<tr>
<td>( \hat{K}_\infty )</td>
<td>0.17829</td>
<td>1.3999</td>
<td>2.92277</td>
</tr>
<tr>
<td>( \hat{\chi} )</td>
<td>-0.5000</td>
<td>-0.53048</td>
<td>-0.5004</td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>5.0000</td>
<td>5.1981</td>
<td>4.9995</td>
</tr>
</tbody>
</table>
The modal Iwan model was then used to predict the response of the system to each of the impulsive forces that were used in the experimental simulation (15 for the 1st mode, 6 for the 2nd and 3rd modes). Again, the modal simulations were integrated using the average acceleration Newmark-Beta integration scheme with an iterative Newton loop to solve the residual modal force equation. The procedure described in Sec. 5.1, was also used to find the energy dissipation and frequency of the modal Iwan simulated data.

7.1.4. Academic Results

A comparison was made between the discrete Iwan simulations and the modal Iwan simulations to see if the deduced modal models could represent the academic model with a discrete Iwan joint. Figure 12 shows the modal energy dissipation versus modal force for the 1st mode. The discrete energy dissipation, \( \hat{D}_{\text{Exp}} \) from Eq. (34), was found by applying the procedure described in Sec. 5.1 to the discrete Iwan simulation data from Sec. 7.1.2. The analytical energy dissipation model, \( \hat{D}_{\text{Model}} \) from Eq. (21), was found using the optimized parameters from Table 3. Finally, the modal energy dissipation which is also, \( \hat{D}_{\text{Exp}} \) from Eq. (34), was found by applying the procedure described in Sec. 5.1 to the modal Iwan simulation data from Sec. 7.1.3. The modal Iwan simulations involve integrating each mode separately as a single degree-of-freedom system. Therefore, since the equations of motion are in terms of modal coordinates, no modal filtering was necessary in contrast to the discrete Iwan simulations.
Figure 12: Modal energy dissipation versus modal force for the 1\textsuperscript{st} elastic mode of the academic model.

The comparison for the 1\textsuperscript{st} mode, in Figure 12, reveals that the modal Iwan model does an excellent job of predicting the energy dissipation of the real nonlinear system with one discrete Iwan joint over a range of impact loads that span the micro-slip and macro-slip regions. Also, the analytical model gives a good approximation to the modal Iwan simulation except right at the transition to macro-slip. Recall that the analytical model is an estimate to the actual dissipated energy and is shown to have a more discontinuous nature than the both the integrated models. Both the discrete and modal models show a more gradual ascension into the macro-slip region. In addition to the energy dissipation, the natural frequency of the 1\textsuperscript{st} mode is compared in Figure 13.
Figure 13: Frequency versus modal force for the 1st elastic mode of the academic model.

Figure 13 plots the natural frequency of the 1st mode versus the modal force. The discrete natural frequency, $\hat{f}_{Exp}$ from Eq. (35), was found by applying the procedure described in Sec. 5.1 to the discrete Iwan simulation data from Sec. 7.1.2. The analytical natural frequency model, $\hat{f}_{Model}$ from Eq. (26), was found using the optimized parameters from Table 3. Finally, the modal natural frequency which is also, $\hat{f}_{Exp}$ from Eq. (35), was found by applying the procedure described in Sec. 5.1 to the modal Iwan simulation data from Sec. 7.1.3. Again, the modal Iwan simulations involve integrating each mode separately as a single degree-of-freedom system, and no modal filtering was necessary. The comparison for the 1st mode, reveals that the modal Iwan model again does a very good job of predicting the natural frequency of the real nonlinear system with one discrete Iwan joint for a range of impact loads that span the micro-slip and macro-slip regions. The analytical model gives a good approximation to the modal Iwan simulation except right at the transition
to macro-slip. The optimized model is shown to agree with both the integrated models over the entire range of the micro-slip region and once the modal force is well into the macro-slip region. Also, note that the change in frequency or stiffness of the first modal Iwan model is small due to the low $\hat{K}_r$ value.

![modal energy dissipation per cycle versus modal force for the 2nd elastic mode of the academic model](image.png)

**Figure 14:** Modal energy dissipation per cycle versus modal force for the 2nd elastic mode of the academic model.

The comparison for the 2nd mode, in Figure 14, reveals that the modal Iwan model does a very good job of predicting the energy dissipation for a range of impact loads that span the micro-slip and macro-slip regions. The analytical model appears to have deduced a slightly higher slip force, $\hat{F}_s$, than the discrete simulation. Yet, the modal model seems to slip at a similar force as the discrete simulation. In comparison to Figure 12, note that the level of energy dissipated by the second mode is approximately the same as the first mode. This may be misleading since the 2nd mode will contain more cycles in a given span of time than the 1st mode.
Figure 15: Frequency versus modal force for the 2nd elastic mode of the academic model.

Figure 15 plots the natural frequency of the 2nd mode versus the modal force. The comparison for the 2nd mode, again reveals that the modal Iwan model is exceptional at predicting the system with a discrete Iwan joint. Also, note that the change in frequency or stiffness of the second modal Iwan model is larger than the change in frequency of the first mode due to the different $\hat{K}_r$ values.
Figure 16: Energy dissipation per cycle versus modal force for the 2nd elastic mode of the academic model.

The comparison for the 3rd mode, in Figure 16, reveals that the modal Iwan model simulations agree with the discrete modal Iwan simulations. Again, comparing with Figure 12 and Figure 14, note that the level of energy dissipated by the third mode is approximately the same as the first two modes; however, the 3rd mode will contain more cycles in a given span of time.
Figure 17: Frequency versus modal force for the 3rd elastic mode of the academic model.

Figure 17 plots the natural frequency of the 3rd mode versus the modal force. The 3rd modal Iwan model is similar to the result seen from the 2nd mode.

7.1.5. Discussion

All the optimized modal Iwan models seem to predict the discrete Iwan simulations in terms of the objective functions (energy dissipation and stiffness). The energy dissipation per cycle for all of the modes is similar; however, this may be misleading in view of the damping of the system since the higher modes will contain more cycles in a given span of time.

7.2. Finite Element Simulations

Both the discrete Iwan model and the modal Iwan model have been implemented into Sierra/SD (Salinas), a structural dynamics finite element code developed by Sandia National
Laboratories [16, 17]. Similar to the academic model, Sierra/SD was used to simulate the response of a structure with several discrete Iwan models, in order to generate experimental data that was then fit to a modal Iwan model. Then the response of the modal Iwan model could be compared to that of the simulated experimental model to evaluate the proposed procedure.

7.2.1. Finite Element Model

The model of interest is a beam with a link attached to the center of the beam. The general dimensions of the beam (20" x 2" x 0.25") and link (3.5" x 0.5" x 0.125") are shown in Figure 18. The beam is meshed with fairly course hexahedron elements (approximate edge length varies between 0.125" and 0.25") with a total of 1,554 nodes to keep the computational cost reasonable. The first five natural frequencies of this model were compared to the natural frequencies of a finely meshed model containing 37,553 nodes, and it was found that the error was less than 1%.

Figure 18: Computational model of a beam with several discrete Iwan joints.
The representative model has four square washers in contact with the beam as seen in Figure 18. Four discrete Iwan models are defined for the two in-plane shear directions at the contact patches between the washers and the beam. Note that no other damping is included in this model, so for the discrete Iwan simulations, all of the energy dissipation is due to the Iwan model.

![Diagram showing location of discrete Iwan models](image)

**Figure 19:** Location of discrete Iwan models. An Iwan model is used at each surface where the washers come into contact with the beam.

In addition to the Iwan elements defined in the in-plane directions, constraints are placed on all out-of-plane rotations and displacements to prevent penetration and separation between the washers and the beam. Two of the washers are merged to the link which adds stiffness to the beam. The beam, link, and washers are given the linear elastic material properties of Stainless Steel 304.

### 7.2.2. Discrete Iwan (Experimental) Simulations

An experiment was simulated by exciting the finite element model with an impulsive force. The force was applied to nine nodes at the tip of the beam as shown in Figure 18. A half sine forcing function was used with a duration of 1.6 ms. The duration of the force was chosen to primarily excite the first three elastic modes of vibration, which will be analyzed in this thesis. Note that the same force function is used for all the simulations in contrast with
the academic model simulations. Nine impacts, with amplitudes of 1, 5, 10, 50, 100, 500, 1000, 5000, and 10000lbf, were used to excite the structure in both the micro-slip and macro-slip regions.

The acceleration time histories at all nodes were extracted and filtered using the pseudo inverse of the mode shape matrix including the first 50 mode shapes from a linear Eigen analysis. This was accomplished using a modal filter [20], i.e. by approximating the response as follows,

\[ \ddot{x} = \Phi \ddot{q} \]  

(43)

where \( \Phi \) is the a subset of the mode shape matrix that includes the first 50 linear modes, and then pre-multiplying both sides of Eq. (43) with the pseudo-inverse of the mode shape matrix. The parameters for all four discrete Iwan elements are shown in Table 4.

**Table 4: Iwan parameters used for the discrete Iwan model in the FE simulations.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_\chi )</td>
<td>300 lbs</td>
</tr>
<tr>
<td>( K_T )</td>
<td>( 2.75 \times 10^5 ) lbs/in</td>
</tr>
<tr>
<td>( \chi )</td>
<td>-0.3</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The discrete Iwan parameters used for these simulations were those found from previous experimental studies of a lap joint [11].

7.2.3. **Modal Iwan Simulations**

The methodology described in Sec. 6.2 was then used to deduce the modal Iwan parameters of the first three elastic bending modes. The optimized parameters for the first three modes of the finite element (FE) model are shown in Table 5.
Table 5: Iwan parameters used for the modal Iwan model for the FE simulations.

<table>
<thead>
<tr>
<th>Modal Iwan Parameters</th>
<th>1st Mode Optimized Values</th>
<th>2nd Mode Optimized Values</th>
<th>3rd Mode Optimized Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{F}_S$</td>
<td>197.2</td>
<td>761.2</td>
<td>625.7</td>
</tr>
<tr>
<td>$\hat{K}_R$</td>
<td>$4.69\cdot10^4$</td>
<td>$2.01\cdot10^4$</td>
<td>$9.21\cdot10^5$</td>
</tr>
<tr>
<td>$\hat{K}_\infty$</td>
<td>$6.09\cdot10^5$</td>
<td>$4.70\cdot10^6$</td>
<td>$1.77\cdot10^7$</td>
</tr>
<tr>
<td>$\hat{\chi}$</td>
<td>-0.284</td>
<td>-0.9998</td>
<td>-0.430</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>0.019</td>
<td>0.107</td>
<td>$7.45\cdot10^{-5}$</td>
</tr>
<tr>
<td>$f_0$ (Hz)</td>
<td>128.9</td>
<td>345.8</td>
<td>688.1</td>
</tr>
</tbody>
</table>

The modal Iwan model was then used to predict the response of the system to the same impulsive force used for the discrete Iwan simulations. This was done using the implementation of the modal Iwan model in the Sierra/SD finite element code. Hence, it was fairly straightforward to obtain the response of the modal model at every node of the finite element model. The modal coordinate amplitudes were not provided directly, so they were estimated from the acceleration time histories using the same process described in Sec. 7.2.2. The step size of the integrator was set to $1\cdot10^{-5}$ seconds with 20,000 time steps for a total time history of 0.2 seconds.

7.2.4. Comparison of Modal Time Responses

Now that the modal model has been identified (Table 5), one can compare its response to the simulated experiment with a discrete number of Iwan elements. For an impact force of 50lbf, the discrete and modal responses are shown in Figure 20.
The modal velocity of the discrete and modal simulations are shown to match quite well for the first three elastic modes. For the 1\textsuperscript{st} and 3\textsuperscript{rd} modes, the modal velocity ring-downs are shown to dissipate a noticeable amount of energy over the time period plotted, while the 2\textsuperscript{nd} mode does not show much decay. The discrete simulations of the 1\textsuperscript{st} and 3\textsuperscript{rd} modes seem to damp the response more than the modal Iwan simulations. After comparing the discrete and modal responses for several other force levels, it becomes clear that one would have to compare a large number of time histories to assess the performance of the

Figure 20: Comparison of the response of the first three elastic modes for both the modal and discrete Iwan simulations for an impact force of 50lbf.
modal Iwan model over a range of impact forces. It is more effective to compare the modal energy dissipation and frequency of each model over the entire range of modal force.

**7.2.5. Comparison of Modal Energy Dissipation and Frequency**

A comparison was made between the discrete Iwan simulations and the modal Iwan simulations to see if the deduced modal models could represent the simulated discrete Iwan experiment with respect to the modal energy dissipation and frequency. Figure 21 shows the modal energy dissipation versus modal force for the 1st mode. The discrete energy dissipation, $\hat{D}_{Exp}$ from Eq. (34), was found by applying the procedure described in Sec. 5.1 to the discrete Iwan simulation data from Sec. 7.2.2. The analytical energy dissipation model, $\hat{D}_{Model}$ from Eq. (21), was found using the optimized parameters from Table 5. Finally, the modal energy dissipation that is also $\hat{D}_{Exp}$ from Eq. (34), was found by applying the procedure described in Sec. 5.1 to the modal Iwan simulation data from Sec. 7.2.3. The modal Iwan simulations involve integrating each mode separately as a single degree-of-freedom system. Therefore, since the equations of motion are in terms of modal coordinates, no modal filtering was necessary in contrast to the discrete Iwan simulations.
Figure 21: Modal energy dissipation versus modal force for the 1st elastic mode. The black lines show experimental simulations of the finite element truth model (with discrete Iwan joints) and the red dashed line shows the dissipation for the modal Iwan model. The two compare well for a range of impact forces from 1lbf to 10,000lbf.

The comparison for the 1st elastic mode, in Figure 21, reveals that the modal Iwan model predicts the energy dissipation for a range of impact loads that span the micro-slip and macro-slip regions. However, the data at higher force levels shows disagreement between the discrete and modal simulations. Based on Figure 21, one would expect the modal Iwan model to over-predict the damping in the actual discrete system at high force levels. Notice that each impact force level is plotted above the each modal energy dissipation curve.
Figure 22: Natural frequency versus modal force for the 1st elastic mode of the FE model. The frequencies also compare well for a range of impact forces from 1 lbf to 10,000 lbf.

Figure 22 plots the natural frequency of the 1st mode versus the modal force. The discrete natural frequency, $\hat{f}_{Exp}$ from Eq. (35), was found by applying the procedure described in Sec. 5.1 to the discrete Iwan simulation data from Sec. 7.2.2. The analytical natural frequency model, $\hat{f}_{Model}$ from Eq. (26), was found using the optimized parameters from Table 3. Finally, the modal natural frequency, which is also $\hat{f}_{Exp}$ from Eq. (35), was found by applying the procedure described in Sec. 5.1 to the modal Iwan simulation data from Sec. 7.2.3. Again, the modal Iwan simulations involve integrating each mode separately as a single degree-of-freedom system, and no modal filtering was necessary. The comparison for the 1st mode reveals that the modal Iwan simulations and analytical model shows good agreement when the system is in far into the micro-slip or macro-slip regions. However, disagreement is seen in the transition region from micro-slip to macro-slip.
The comparison for the 2nd elastic mode, in Figure 23, reveals that the modal Iwan model does a fine job of predicting the energy dissipation for a range of impact loads. The slope of the modal Iwan model in Figure 23 has a value of approximately 2 corresponding to a $\hat{\chi}$ value of -0.999 from Table 5, which is characteristic of a linearly damped system. Thus, the dissipation in this mode could be modeled with a linear mode and a linear modal damping ratio instead of a modal Iwan model. Apparently, the location of the link is such that the discrete Iwan joints are not exercised by this mode. The energy dissipation is thought to be from algorithmic damping in the integrations scheme used in the finite element program Salinas.
Figure 24: Natural frequency versus modal force for the 2nd elastic mode of the FE model. The 2nd bending mode does not reach the macro-slip region.

Figure 24 plots the natural frequency of the 2nd mode versus the modal force. The comparison for the 2nd mode reveals that the frequency does not suddenly drop meaning the 2nd mode has not reached the macro-slip region. In addition, the slight shift in frequency is almost unnoticeable, meaning the Iwan joint has not lost a lot of stiffness over this force range.
Figure 25: Modal energy dissipation versus modal force for the 3rd elastic mode. The modal simulations agree well with the discrete simulations for this mode in the micro-slip region, but not as well in the macro-slip region.

The comparison in Figure 25 shows that the deduced modal Iwan model for the 3rd elastic mode predicts the energy dissipation in the micro-slip region, for which a $\hat{\chi}$ value of $\hat{\chi}_3 = -0.43$ was obtained. However, in this region the analytical model and modal simulation seem to over predict the energy dissipation. In addition, for the macro-slip region, the modal Iwan model appears to under predict the energy dissipation. The slope of the discrete Iwan simulations seems to be greater than the slope of the modal Iwan model in the macro-slip regime. This suggests that Eq. (21) may not entirely describe the energy dissipation in the macro-slip region for this system which includes multiple discrete Iwan models. For this mode, it is thought that the transition from micro-slip to the macro-slip region is spread out over a larger range of forces due to different Iwan joints slipping at different force levels.
Figure 26 plots the natural frequency of the 3rd mode versus the modal force. The 3rd modal Iwan model is similar to the result seen from the 1st mode; however, the simulated discrete data is a bit messier.

7.2.6. Validating the Modal Model

In order to compare the discrete and modal responses in the time domain, a modal filter was applied to the discrete data to only include the contribution of the first three bending modes. Note that including the higher elastic modes did not change the response of the discrete simulation dramatically, but some higher frequency response was noticed. The model is assumed to be accurate for a bandwidth that spans the first three modes with a frequency range of approximately 10 to 700 Hz. The filtered discrete response was compared to the modal simulated response in the time domain as shown in Figure 27.
The velocity in the z-direction at the midpoint of the finite element model is plotted in Figure 18. The velocity ring-down for the modal model (dashed line) is observed to compare very well with the ring-down of the filtered discrete model (solid line).

### 7.2.6.1. Discussion

In comparison with the academic modal simulations, the finite element modal simulations are slightly worse at predicting the system with multiple discrete Iwan joints. However, the results seem to validate the use of a modal Iwan model to capture the effect of discrete Iwan joints in the micro-slip region. The modal Iwan model may need to be modified to capture the response of a system at higher force levels corresponding to the macro-slip region.
8. Beam Experiments

The modal Iwan methodology was also assessed experimentally using experimental measurements from two free-free bolted structures. A one beam structure with a bolted link attachment, similar to the finite element model in Sec. 7.2.1, was tested first. The second structure tested consisted of two beams bolted together. The experimental setup was designed to minimize the effect of damping associated with the boundary conditions. Free boundary conditions were used because any other choice, e.g. clamped, would add significant damping to the system.

8.1. One Beam Mid-Link Test Structure

One of the structures tested in this thesis was first used by Sumali in [26]. The beam was designed to have the following characteristics: the structure had numerous modes from 0-2000 Hz, the modes are well separated in frequency and the modes do not switch order when the link is attached. Previously, a washer was inserted between the link and the beam as was done for the finite element model in [26]. Initial tests were done in this configuration but the damping in the joint seemed linear, indicating that there was no slip present in the joint for the force levels tested. The washers were then removed in order to spread out the clamping force between the beam and the connecting element. The resulting configuration is shown in Figure 28. Also, the bolt torque for the original experiments was set between 80 and 110 inch-pounds which results in bolt preload force of approximately 1600-2200lbf. For reference, the Society of Automotive Engineers (SAE) provides the general torque specification for this type of bolt to be approximately 75.0 in-lbs [27] which results in a bolt preload force of approximately 1500lbf. For this work, the bolts were tightened to torque
levels of only 5, 10, and 100 inch-lbs. These changes were made in a deliberate attempt to cause the joint to slip more easily; however, as will be discussed, the bolted connections did not contribute much observable damping to the system.

The dimensions of the beam and link are the same as in the finite element model described in Sec. 7.2.1. However, there is no washer that separates the beam (20'' x 2'' x 0.25'') from the link (3.5'' x 0.5'' x 0.125'') as shown in Figure 28. The bolts used to attach the link to the beam were 1/4''-28 fine-threaded bolts. All components were made of AISI 304 stainless steel.

Figure 28: Schematic of the one beam mid-link test structure connection that was used in the experimental setup.
8.2. **One Beam Mid-Link Damping Ratio Comparison**

A comparison was done to ensure that the addition of a small bolted attachment would increase the damping in the structure. A monolithic structure, without interfaces and bolts, was chosen for comparison to ensure that the measured damping was only due to the structure itself and the support conditions. The same approximate free-free boundary conditions, which are discussed in Sec. 8.4, were used for the beam with and without the link attachment. The damping ratios for the first seven modes were found using the Algorithm of Mode Isolation (AMI) [29, 30] and are presented in Table 6.

**Table 6: Averaged Modal Damping Ratios for the one beam mid-link test structure.**

<table>
<thead>
<tr>
<th>Elastic Mode #</th>
<th>5 in-lbs Torque, $\zeta$ (%)</th>
<th>10 in-lbs Torque, $\zeta$ (%)</th>
<th>100 in-lbs Torque, $\zeta$ (%)</th>
<th>Monolithic Structure, $\zeta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>0.036</td>
<td>0.021</td>
<td>0.016</td>
</tr>
<tr>
<td>2</td>
<td>0.060</td>
<td>0.060</td>
<td>0.058</td>
<td>0.057</td>
</tr>
<tr>
<td>3</td>
<td>0.064</td>
<td>0.071</td>
<td>0.052</td>
<td>0.044</td>
</tr>
<tr>
<td>4</td>
<td>0.099</td>
<td>0.097</td>
<td>0.093</td>
<td>0.082</td>
</tr>
<tr>
<td>5</td>
<td>0.032</td>
<td>0.034</td>
<td>0.029</td>
<td>0.020</td>
</tr>
<tr>
<td>6</td>
<td>0.018</td>
<td>0.017</td>
<td>0.017</td>
<td>0.016</td>
</tr>
<tr>
<td>7</td>
<td>0.040</td>
<td>0.046</td>
<td>0.033</td>
<td>0.025</td>
</tr>
</tbody>
</table>

For the one beam structure, similarities are shown between the damping ratios of the 5 and 10 in-lbs torques. This is to be expected with similar bolt torque levels. Also, the damping ratios of the 5 and 10 in-lbs torques tend to be slightly higher than the 100 in-lbs torque, which means that by loosening the bolt torques the joint was able to slip more easily. More importantly, similar damping ratios are seen between the 100 in-lbs torque and the monolithic structure. Thus, with a large bolt torque, the one beam mid-link structure has very little measurable damping from the bolted joints, and most of the measured damping, as with the monolithic structure, comes from the suspension setup and material damping.
Therefore, since the one beam mid-link test structure had very little detectable nonlinear damping, the two beam test structure will be the focus of the experimental results section. The results for the one beam mid-link test structure can be found in Appendix A.

8.3. Two Beam Test Structure

The two beams test structure consists of two beams bolted together with four bolts as shown in Figure 29. The two beams, each with dimensions $20" \times 2" \times 0.25"$ were fastened together with 1/4"-28 fine-threaded bolts and all components were made of AISI 304 stainless steel. The bolts were tightened to three different torque levels in these tests: 10, 30, and 50 in-lbs. For reference, the Society of Automotive Engineers (SAE) provides the general torque specification for this type of bolt to be approximately 75.0 in-lbs [27] which results in bolt preload force of approximately 1500lbf. The largest torque used here was somewhat lower than this specification, but, as with the one beam test structure, this structure became quite linear for the range of excitation forces that were practical with this setup, so the bolts were kept somewhat loose to accentuate the nonlinearity. In the future, experimental work should explore methods of exciting the structure with higher force levels (closer to what might be seen in the applications of interest) so that more realistic torques can be used.
8.4. Experimental Setup

The dynamic response of the both test structures was captured using a scanning laser Doppler vibrometer (Polytec PSV-400), which measured the response at 70 points on the two beam structure. In addition, a single point laser vibrometer (Polytec OFV-534) was used to measure at a reference point to verify that the hammer hits were consistent. The reference laser was positioned close to the impact force location as seen in Figure 30.
Figure 30: Photograph of the suspension setup for the two beam test structure.

In this test setup, the structure is suspended by 2 strings that support the weight of the structure and 8 bungee cords which prevent excessive rigid-body motion. The bungees and strings were connected to the beam at locations where the odd bending modes have little motion in order to minimize the damping added to the system for these modes.

An Alta Solutions automated impact hammer with a nylon hammer tip was used to supply the impact force, which is measured by a force gauge attached between the hammer and the hammer tip. Additional measurements were taken at higher force levels using a modal hammer; however, the supplied impact force was not as consistent. The mean and standard deviation of the maximum impact force for all of the torque levels and force levels that were used in this study are shown in Table 7.
Table 7: Mean and standard deviation of the maximum impact force for all 70 measurements of the two beam structure.

<table>
<thead>
<tr>
<th>Torque (in-lbs)</th>
<th>Hammer Level (1 lowest - 4 highest)</th>
<th>Mean Impact Force (N)</th>
<th>Standard Deviation of Impact Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>20.2</td>
<td>0.80</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>32.8</td>
<td>0.27</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>86.4</td>
<td>0.68</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>288.6</td>
<td>6.10</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>24.1</td>
<td>0.38</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>30.9</td>
<td>0.51</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>52.8</td>
<td>3.84</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>180.1</td>
<td>58.24</td>
</tr>
<tr>
<td>30</td>
<td>Modal Hammer</td>
<td>1444.5</td>
<td>139.34</td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>20.8</td>
<td>0.44</td>
</tr>
<tr>
<td>50</td>
<td>2</td>
<td>36.5</td>
<td>0.28</td>
</tr>
<tr>
<td>50</td>
<td>3</td>
<td>60.3</td>
<td>0.61</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
<td>238.6</td>
<td>15.30</td>
</tr>
<tr>
<td>50</td>
<td>Modal Hammer</td>
<td>1392.1</td>
<td>172.48</td>
</tr>
</tbody>
</table>

The automatic hammer provides a range of force levels between approximately 20 and 300 N. However, the force level is dependent upon the distance between the hammer tip and the beam and the voltage supplied to the automatic hammer. For these reasons, the impact force varies for each measurement. For the automatic hammer, the standard deviation tends to increase as the force level is increased. At the highest force level, the automatic hammer has a large spread for all the torque levels especially the 30 in-lbs torque. The modal hammer is able to reach much higher force levels (approximately 1400 N); however, the standard deviations are much larger when compared to the automatic hammer.

8.5. Lab Setup Challenges

The damping ratios of a freely supported structure are sensitive to the support conditions, as was explored in detail by Carne, Griffith, and Casias in [28]. Therefore, special attention must be given to the support conditions to assure that the damping that they
add does not contaminate the results. Initially, the beam structures were suspended by two strings that act as pendulum supports as was done in [28]. These support conditions contributed very little damping to the system; however, several obstacles were encountered with that setup.

Specifically, the velocity of the beam was measured with a scanning laser Doppler vibrometer in order to eliminate any damping associated with the cables that must be added if accelerometers were used. Hence, if the beam swings significantly in its pendulum mode, the point which the laser is measuring may change significantly during the measurement. Also, an automated hammer was used to excite the beam, but the hammer only retracts about 1 inch after impact. As a result, the pendulum motion of the beam caused almost unavoidable double hits when the bungee cords were not present. Finally, in the processing described subsequently, it is important for the automatic hammer to apply a highly consistent impact force. Any ambient swinging of the beam caused the impact forces to vary from test to test. When the bungee cords were not present, it was extremely difficult and time consuming to try to manually eliminate the ambient swinging. For these reasons, eight soft bungee cords were added to the setup to suppress the rigid body motion of the beam while attempting to add as little stiffness and damping as possible to the system. The final setup was similar to that used in [26] and is shown in Figure 30. This setup was used for all of the measurements shown in this paper.

A comparison was done to ensure that the addition of bungee cords did not add significant damping to the system. The same monolithic structure from Table 6 was chosen to ensure that the measured damping was only due to the structure itself and the support conditions. A single stainless steel beam was suspended with two strings with and without
the bungees cords and the damping ratios for the first three modes were found using the Algorithm of Mode Isolation (AMI) [29, 30].

**Table 8: Averaged modal damping ratios for a single beam with and without bungees.**

<table>
<thead>
<tr>
<th>Elastic Mode #</th>
<th>ζ without bungees (%)</th>
<th>ζ with bungees (%)</th>
<th>Percent Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.010</td>
<td>0.016</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>0.025</td>
<td>0.057</td>
<td>128</td>
</tr>
<tr>
<td>3</td>
<td>0.020</td>
<td>0.044</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 8 shows the damping of all of the modes is very light, as one would expect for a monolithic structure. When the bungees were added to the setup, the damping ratios for all modes increased from 60 to 128 percent. The bungees and strings were connected to the beam at locations where the motion of the symmetric or odd bending modes is minimum. These locations are expected to add some damping to the second mode; however, the results show that the supports added damping to the first and third modes as well. These damping ratios from Table 8 are averaged over a range of force levels; the damping of the monolithic structure should behave linearly, so the force level did not vary the damping significantly.

The damping in the monolithic structure presumably comes from material damping and the damping provided by the support conditions. For comparison, the two beam test specimen, presented in Sec. 8.3, was curve fit to estimate the best fit linear modal damping ratios at various torque levels and the results are presented in Table 9. Due to the nonlinearity introduced by the joints in the test specimen, the damping ratios seem to change with the amount of excitation applied. The damping ratios presented in Table 9 are an average over all of the data from a range of force levels, and hence they represent a linear fit to a structure which is known to be nonlinear and this probably does introduce some distortion.
Table 9: Averaged modal damping ratios for the two beam test structure.

<table>
<thead>
<tr>
<th>Two Beam Test Structure</th>
<th>10 in-lbs Torque, $\zeta$ (%)</th>
<th>30 in-lbs Torque, $\zeta$ (%)</th>
<th>50 in-lbs Torque, $\zeta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elast. Mode #</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1.2</td>
<td>0.29</td>
<td>0.16</td>
</tr>
<tr>
<td>2</td>
<td>0.57</td>
<td>0.48</td>
<td>0.26</td>
</tr>
<tr>
<td>3</td>
<td>0.31</td>
<td>0.16</td>
<td>0.11</td>
</tr>
</tbody>
</table>

In general, the damping is observed to decrease as the bolt torque is increased. This was expected since increasing the bolt torque inhibits micro-slip and hence should decrease the measured damping, although occasionally the opposite has been observed for certain modes [19]. However, even at the tightest bolt torque (50 in-lbs) the modal damping ratios are significantly larger than those of the monolithic structure, by factors of 10, 4.5, and 2.5 for the first three modes respectively. Although the two beam structures mass and geometry is different than the monolithic one beam structure, it seems that a significant portion of the measured damping is due to the joints in this structure.

8.6. Lab Data Processing

Two approaches were explored to extract modal velocity ring-downs from the laboratory data. First, mass normalized mode shapes were found by fitting a linear modal model with the Algorithm of Mode Isolation (AMI) [29, 30]. Then the mode shapes were used in a modal filter as was done in Sec. 7.2.2 with Eq. (43). However, when using a modal filter the modal responses showed clear evidence of frequency content due to other modes, which would contaminate the Hilbert transform analysis from Sec. 5.1. Since both system's modes are well separated, the modes were instead isolated by creating a band pass filter to filter a single mode, as was done in [19], using a fourth order Butterworth filter. The filtered
responses were then divided by the corresponding mass normalized mode shape at each point, \( j \), to estimate the modal displacement as,

\[
\dot{q}_r = \frac{\dot{x}_j}{\Phi_{jr}}
\]  

(44)

where \( \Phi_{jr} \) is the mass normalized mode shape value at the measured point \( j \).

The experimental mass normalized mode shapes for the first six elastic modes are shown in Figure 31 when the bolts are tightened to 30 in-lbs. These mode shapes of the two beam structure were found at the lowest force level of the automatic hammer.
Figure 31: Two Beam mass normalized modes shapes at 30 in-lbs torque.

For the two beam structure, there were 70 measurement points which were then averaged to estimate a single modal velocity for the first three elastic modes in Figure 31. Some measurement points were excluded from averaging process if the mode was excited too heavily or not sufficiently. A trimmed mean was used to determine which measurements to keep. The trimmed mean procedure excluded 8 high and low outliers from the set of 70 measurements points. All measurement points whose maximum velocity was within 50
percent of the trimmed mean were kept. The resulting statistics on the filtered impact hammer data are presented Table 10.

**Table 10: Mean and standard deviation of the maximum impact force for the set of measurements that was used.**

<table>
<thead>
<tr>
<th>Torque (in-lbs)</th>
<th>Hammer Level (1 lowest - 4 highest)</th>
<th>Mean Impact Force (N)</th>
<th>Standard Deviation of Impact Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>20.0</td>
<td>0.088</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>32.8</td>
<td>0.025</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>86.5</td>
<td>0.041</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>289.3</td>
<td>0.213</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>24.2</td>
<td>0.013</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>30.8</td>
<td>0.019</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>52.7</td>
<td>0.125</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>191.3</td>
<td>1.585</td>
</tr>
<tr>
<td>30</td>
<td>Modal Hammer</td>
<td>1475.7</td>
<td>3.081</td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>20.9</td>
<td>0.009</td>
</tr>
<tr>
<td>50</td>
<td>2</td>
<td>36.5</td>
<td>0.005</td>
</tr>
<tr>
<td>50</td>
<td>3</td>
<td>60.3</td>
<td>0.011</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
<td>237.2</td>
<td>0.310</td>
</tr>
<tr>
<td>50</td>
<td>Modal Hammer</td>
<td>1400.4</td>
<td>3.225</td>
</tr>
</tbody>
</table>

All of the filtered standard deviations in Table 10 are smaller than the initial standard deviations shown in Table 7. Again, for the automatic hammer, the standard deviation tends to increase as the force level is increased. Yet, even at the highest force level, the automatic hammer has a much more reasonable maximum standard deviation of 1.6 N and the modal hammer has a standard deviation of approximately 3 N.

The automatic hammer measurement set with the highest standard deviation (1.6 N) is at a torque level of 30 in-lbs with the highest automatic hammer level. In this case, the 70 measurement points were cut down to 47 measurement points using the trimmed mean approach described previously. The filtering process for the third mode of this reduced measurement set is shown in Figure 32 and Figure 33.
The cut-off frequencies used for the band pass filter of the third mode are 450 and 495 Hz. The fast Fourier Transform (FFT) of the resulting filtered velocity is shown in Figure 32. After the band pass filtering process, the measurements at each point were scaled by the corresponding mass normalized mode shape value, Eq. (44), in order to obtain a single average modal velocity for each mode from the set of 47 measurement points. The resulting time responses are show in Figure 33, as well as the average time response that was used to identify a modal Iwan model.
The zoomed in view shows that the kept set of measurements were fairly consistent.

The averaged response is used to find the energy dissipation and frequency for each mode of interest. Therefore, it is important that the averaged response follows the decay envelope and has the same frequency content of all the filtered responses. Even for the measurement set with the one of the largest standard deviations, all the responses are adequately captured by the average modal time response. For the other sets of measurements with smaller standard deviations, the responses are even more consistent.

8.7. Two Beam Results

The two beam bolted structure was tightened to 30 in-lbs of torque, and the measurements at all the force levels were band-pass filtered and averaged as described previously to isolate the first bending mode. The optimization procedure was then used to
find the modal parameters that best fit the experimental data to the modal Iwan models presented in Sec. 4.1 and 4.2. The model without the viscous damper relies entirely on the Iwan joint to dissipate energy as opposed to the Iwan model with a viscous damper. The parameters of the optimized modal models are shown in Table 11 for the first bending mode.

**Table 11: Optimized parameters of the first bending mode of vibration at a bolt torque of 30 in-lbs, for the modal models with and without the viscous damper.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Modal Iwan Model</th>
<th>Modal Iwan Model with a Viscous Damper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{F}_s$</td>
<td>6.23</td>
<td>2.33</td>
</tr>
<tr>
<td>$\hat{K}_r$</td>
<td>$2.61 \cdot 10^5$</td>
<td>$1.37 \cdot 10^5$</td>
</tr>
<tr>
<td>$\hat{K}_\infty$</td>
<td>$3.19 \cdot 10^5$</td>
<td>$4.41 \cdot 10^5$</td>
</tr>
<tr>
<td>$\hat{\chi}$</td>
<td>-0.272</td>
<td>-0.178</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>0.836</td>
<td>0.0316</td>
</tr>
<tr>
<td>$\hat{C}$</td>
<td>N/A</td>
<td>3.96</td>
</tr>
</tbody>
</table>

Table 11 shows that by adding a viscous damper, all of the best fit modal Iwan parameters have changed. One limitation of only using the modal Iwan model is that the energy dissipation, Eq. (21), and the frequency, Eq. (26), are both dependent upon the modal Iwan parameters $\{\hat{F}_s, \hat{K}_r, \hat{\chi}, \hat{\beta}\}$. Therefore, during the optimization process the two objective functions, $f_D$ and $f_f$, are unable to be modified independently. The addition of a viscous damper, $\hat{C}$, enables the energy dissipation objective function, $f_D$, to be modified independently, thus providing a more versatile model to fit the experimental measurements.
Figure 34: Comparison between measured natural frequency versus force and the two modal joint models.

Figure 34 shows the natural frequency of the modal Iwan model versus the total modal force for the two modal models, reconstructed using Eq. (22). The measurements show that the natural frequency of this mode changes approximately 7 Hz over the range of forces that were applied. Both models seem to be capable of capturing the change in natural frequency over this range. Unfortunately, the natural frequency is not observed to level off at a frequency $\omega_\infty$ as predicted by theory. This suggests that the system never completely reaches macro-slip or that macro-slip is over before the Hilbert transform algorithm is able to capture the macro-slip frequency, making it difficult to estimate the parameters $\{\hat{F}_s, \hat{K}_f, \hat{K}_\infty\}$. 
Note that the optimized models have identified a value for the slip force, $\hat{F}_s$, that is in the range of the measured forces. Thus, at the highest measured force levels macro-slip has been initiated in both models. Unfortunately, the exciter that was used was not capable of even higher forces so macro-slip could not be fully characterized.

**Figure 35:** Energy dissipation comparison of two optimized modal models to experimental data over a range of forces.

Figure 35 shows the modal energy dissipation versus total modal force for the two modal models and the experimental data at five different excitation levels. The Iwan model without a viscous damper in parallel fails to fit the measurements at low amplitude, while the model with only a viscous damper does not capture the increase in damping at high forces. (Because of the logarithmic scale, the difference at high force levels may appear to be small
yet the damping in the linear model is actually in error by an order of magnitude at high energy.) In contrast, the modal Iwan model with a viscous damper in parallel provides an excellent approximation to the measured energy dissipation. It should also be noted that the disagreement between the Iwan model (without a viscous damper) and the measurement at low force levels is not simply due to the choice of parameters. Considerable effort was spent to optimize this model's parameters to better match the measurements, yet the fit could not be improved without decreasing the agreement of the natural frequency versus force plot in Figure 34. This difficulty disappeared when a viscous damper was added to the model. The differences between these models is more easily visualized by comparing the slope of the energy dissipation versus force curve. As mentioned previously, a single Iwan joint exhibits a slope of $3 + \hat{\ gamma}$ on a log dissipation versus log force plot. Figure 36 compares the slope of the two optimized modal models with the experimentally measured slope. A fifth order polynomial was fit to the laboratory data in order to compute its slope. Without an additional viscous damper, the modal Iwan model has a much larger slope than the laboratory data at low force levels. On the other hand, when a viscous damper is added in parallel with the Iwan joint, the slope follows the laboratory data more closely over the entire range of force levels.
Figure 36: Slope of energy dissipation versus modal force for modal Iwan models and a polynomial fit to the experimental measurements.

This same procedure was repeated for the first three elastic modes at three different bolt torques and the identified modal Iwan parameters are shown in Table 12. Only the first three elastic modes were analyzed in this work due to the lack of response from the higher frequency modes. The last two rows of each section of Table 12 give the natural frequency and modal damping ratio, which can be readily computed from the other parameters.
Table 12: Optimized parameters for a modal Iwan model with a viscous damper. First three elastic modes each at varying bolt torques.

<table>
<thead>
<tr>
<th>Bolt Torque (in-lbs)</th>
<th>1st Elastic Mode</th>
<th>2nd Elastic Mode</th>
<th>3rd Elastic Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>( \hat{F}_s )</td>
<td>0.562</td>
<td>2.33</td>
<td>3.08</td>
</tr>
<tr>
<td>( \hat{K}_T )</td>
<td>1.16·10^5</td>
<td>1.37·10^5</td>
<td>1.35·10^5</td>
</tr>
<tr>
<td>( \hat{K}_\infty )</td>
<td>5.03·10^5</td>
<td>4.41·10^5</td>
<td>4.44·10^5</td>
</tr>
<tr>
<td>( \hat{\chi} )</td>
<td>-0.0237</td>
<td>-0.178</td>
<td>-0.0102</td>
</tr>
<tr>
<td>( \hat{\chi}_{\text{initial}} )</td>
<td>-0.720</td>
<td>-0.871</td>
<td>-0.958</td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>0.0237</td>
<td>0.0316</td>
<td>1.19</td>
</tr>
<tr>
<td>( \hat{\chi} )</td>
<td>1.89</td>
<td>3.96</td>
<td>1.12</td>
</tr>
<tr>
<td>( f_0 ) (Hz)</td>
<td>125.2</td>
<td>121.0</td>
<td>121.1</td>
</tr>
<tr>
<td>( \hat{\zeta} ) (%)</td>
<td>0.120</td>
<td>0.099</td>
<td>0.074</td>
</tr>
</tbody>
</table>

The results above show that the slip force parameter, \( \hat{F}_s \), tends to increase when the bolts are tightened for all modes considered. This is as expected since, as the bolts are tightened, the preload in the bolts increases so larger forces are required to initiate macro-slip. As the bolts are tightened, one would expect that the \( \hat{K}_\infty \) parameter for each mode would stay relatively constant while the joint stiffness, \( \hat{K}_T \), would increase. However, the optimized stiffness parameters, \( \hat{K}_T \) and \( \hat{K}_\infty \), seem not to follow much of a trend for this system. This probably indicates that the measured data is not adequate to reliably estimate \( \hat{K}_\infty \), as might be expected since the excitation force was not sufficient to bring the system
well into macro-slip. The joint parameter, $\hat{\chi}$, can be observed to decrease as the bolt torques are increased from 10 to 30 in-lbs. This is to be expected, since the energy dissipation should resembles a linear system at higher bolt torques. However, from 30 to 50 in-lbs the $\hat{\chi}$ parameter increases. This means the energy dissipation of the Iwan model has become more nonlinear to account for the slip region that contains a great deal of energy dissipation close to the macro-slip region. The $\hat{\chi}_{\text{initial}}$ value is found from Eq. (36) from Sec. 6.1, by fitting a slope to all of the data in the modal energy dissipation versus modal force plot. These values show that as the bolt torque is increased, the nonlinearity of the energy dissipation becomes less and less. Also, for the three modes analyzed, the nonlinear energy dissipation becomes less as the mode number increases. Yet this may not hold true for all modes, as seen from the fast Fourier transforms in Appendix B, the higher frequency modes that were not heavily excited may be more nonlinear than the first three modes analyzed. Finally, the viscous damping parameter, $\hat{C}$, seems to remain in a similar range. The equivalent low-amplitude damping ratio is also shown and these damping ratios are comparable to those in Table 9 and hence they seem to be plausible lower bounds for the damping in the system, due the supports and material damping.

8.8. Validating the Modal Model

The optimized modal model from Table 12 was next validated by comparing the response of the nonlinear model with an experimentally measured response. The response at the midpoint of the 2 beam setup was selected for the location of interest. The bolts of the 2 beam structure were tightened to 30 in-lbs and an impact force with a maximum value of approximately 53 N (or the 3rd force level from Table 10) was applied to the structure using
the automatic hammer. The experimentally measured impact force was used as an input and the modal equations of motion for each of the modes (using the parameters in Table 12) were integrated in time with a Newmark-Beta time integration routine with a Newton-Raphson iteration loop for the nonlinear force in the Iwan model. The response at the midpoint of the beam was then found by adding the contribution of each mode and using the mass normalized mode shapes.

The responses were first compared in the frequency domain where it was easy to ignore the effect of the rigid body modes. In addition, a zeroed early-time fast Fourier transform (ZEFFT) [12] was used to show how the nonlinearity of both the model and the measured data progressed over time. Figure 37 shows the ZEFFTs taken at several different times including: 0.051, 0.29, 0.53, 0.76, and 1.0 second as indicated in the legend. The solid and dashed lines correspond to the experimentally measured response and the simulated response from the modal Iwan models respectively.
Figure 37: ZEFFTs for the midpoint of the structure for both the experimental measurement (solid lines) and the model (dashed lines).

The ZEFFTs show that the first three modes dominate the response in this frequency range and that the frequencies do not shift very much over time. The model matches the measurement very well, except at those frequencies where the measurement falls below the noise floor of the sensors. It is typically necessary to zoom in near each resonance peak to evaluate the ZEFFTs for a system such as this. Figure 37 shows a zoomed in view of the first resonant peak from Figure 38. This comparison reveals that the model agrees quite well with the measurements; both predict a similar variation in the amplitude of the peak with time and a similar level of smearing as the frequency of oscillation increases with time (due to decreasing amplitude). It is important to note that no filtering was performed on the measured data, so this confirms that the filters used when obtaining the modal Iwan parameters have not distorted the data significantly.
Figure 38: Zoomed in view of the first resonant peak with ZEFFTs for the both the experimental measurement (solid lines) and the model (dashed lines).

In order to compare the responses in the time domain, a filter was applied to eliminate the rigid body motion. The measured response was filtered using a fourth order Butterworth filter, with frequencies between 50 and 600 Hz kept. Note that other frequency bands were experimented with, yet including higher frequencies in the filtering process did not change the time response significantly. The filtered measured response was compared to the simulated response in the time domain as shown in Figure 39.
Figure 39: Time response comparison of the filtered experimental measurement (solid lines) and the model (dashed lines).

The velocity ring-down for the model (dashed line) is observed to compare very well with the ring-down of the filtered measurement (solid line). In this type of comparison, it is important to note that the measured data is filtered which could distort the signals, but in this case the distortion would hopefully be minimal since only the rigid body motions have been eliminated.

9. Conclusion

In this thesis, the modal Iwan model was applied to simulation data from an academic or spring mass model and a finite element model with discrete Iwan joints. The 4-parameter Iwan model was used to model the nonlinear damping associated with each mode. This modal model was found to fit the simulation data very well in both the micro-slip region and
the macro-slip region for the academic model. The modal model was also found to fit the finite element simulation data well in the micro-slip region and it agreed somewhat well into macro-slip. This finding is encouraging; the time response of the a collection of modal Iwan models can be found in a tiny fraction of the time required to find the response of the full system in physical coordinates with discrete Iwan joints.

A new modal model, that includes an Iwan model with a viscous damper, was presented to model the experimental modal damping of a free-free structure. This model was found to fit the measurements exceptionally well for the first three bending modes, suggesting that modal coupling was weak and that a modal Iwan model may be an effective way of accounting for the nonlinear damping associated with the mechanical joints of the system. The measurements also showed that it was important to include a viscous damper in parallel with the modal Iwan model in order to account for other sources of experimental energy dissipation associated with the material and the boundary conditions. There are only a few parameters to identify and the parameters $\hat{\chi}$, $\hat{\beta}$, $\hat{C}$ and $\hat{K}_\tau$ are all fairly clearly represented in the modal response. On the other hand, the parameters $\hat{F}_s$ and $\hat{K}_x$ were somewhat difficult to estimate since we were not able to apply large enough input forces to drive the system well into the macro-slip regime. This is likely to always be a problem when impulsive forces are used since the joint dissipates a lot of energy in the first few cycles, before the filters and Hilbert transform have time to stabilize.

The modal Iwan approach is very appealing since it allows one to treat a structure as a set of uncoupled linear modes with slightly nonlinear characteristics in the micro-slip regime. On the other hand, if one were to consider a system level test in physical coordinates with
many joints, each joint would need to be modeled with a discrete Iwan element with a set of unique parameters. First, the full finite element model would need to be integrated at different force levels as opposed to a small set of active modal equations. Second, it is possible that hundreds or even thousands of discrete joint parameters would need to be deduced and optimized which may not be feasible from a set of experimental measurements. In the validation section, the modal Iwan model was used to predict the response of the structure to a measured impulsive force and the comparison showed that the modal Iwan model did accurately predict the measured response over the frequency range of interest. In all cases to date, experimental and analytical results have suggested that this approach can be very successful, except perhaps at very high force levels when the system is well into the macro-slip regime.
10. References


APPENDIX A: One Beam Mid-Link Test Results

The one beam mid-link test results were shown have very little detectable nonlinear damping from Table 9. As was done for the two beam test structure, the mean and standard deviation of the maximum impact force for all of the torque levels and force levels of the one beam mid-link structure are shown in Table 13.

Table 13: Mean and standard deviation of the maximum impact force for all 54 measurements of the one beam mid-link structure.

<table>
<thead>
<tr>
<th>Torque (in-lbs)</th>
<th>Hammer Level (1 lowest - 4 highest)</th>
<th>Mean Impact Force (N)</th>
<th>Standard Deviation of Impact Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>32.4</td>
<td>0.41</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>49.1</td>
<td>5.10</td>
</tr>
<tr>
<td>5</td>
<td>4 (highest)</td>
<td>157.6</td>
<td>1.31</td>
</tr>
<tr>
<td>5</td>
<td>Modal Hammer</td>
<td>1189.0</td>
<td>154.58</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>35.7</td>
<td>0.82</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>63.4</td>
<td>0.82</td>
</tr>
<tr>
<td>10</td>
<td>4 (highest)</td>
<td>165.5</td>
<td>2.24</td>
</tr>
<tr>
<td>10</td>
<td>Modal Hammer</td>
<td>1125.3</td>
<td>152.88</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>30.3</td>
<td>0.60</td>
</tr>
<tr>
<td>100</td>
<td>3</td>
<td>37.1</td>
<td>0.78</td>
</tr>
<tr>
<td>100</td>
<td>4 (highest)</td>
<td>158.1</td>
<td>7.38</td>
</tr>
</tbody>
</table>

For the one beam mid-link structure from Figure 28, the automatic hammer provides a range of force levels between approximately 30 and 160 N. However, the force level is dependent upon the distance between the hammer tip and the beam in addition to the voltage supplied to the automatic hammer. For these reasons, the lowest and highest force varies for each measurement. For the automatic hammer, the standard deviation tends to increase as the force level is increased. The modal hammer is able to reach much higher force levels (approximately 1200 N); however, the standard deviations are much larger when compared to the automatic hammer.
For the one beam structure, there were 54 measurement points which were then averaged to estimate a single modal velocity for each mode. Some measurement points were excluded from averaging process if the mode was excited too heavily or not sufficiently. A trimmed mean was used to determine which measurements to keep. The trimmed mean procedure excluded 6 high and low outliers from the set of 54 measurements points. All measurement points whose maximum velocity was within 50 percent of the trimmed mean were kept. The resulting statistics on the filtered impact hammer data are presented in Table 14.

**Table 14: Mean and standard deviation of the maximum impact force for the reduced set of measurements that was used for the one beam structure.**

<table>
<thead>
<tr>
<th>Torque (N-m)</th>
<th>Hammer Level (1 lowest - 4 highest)</th>
<th>Mean Impact Force (N)</th>
<th>Standard Deviation of Impact Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>32.4</td>
<td>0.021</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>51.0</td>
<td>0.27</td>
</tr>
<tr>
<td>5</td>
<td>4 (highest)</td>
<td>157.4</td>
<td>0.064</td>
</tr>
<tr>
<td>5</td>
<td>Modal Hammer</td>
<td>1198.1</td>
<td>9.60</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>32.4</td>
<td>0.013</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>48.4</td>
<td>0.17</td>
</tr>
<tr>
<td>10</td>
<td>4 (highest)</td>
<td>157.2</td>
<td>0.048</td>
</tr>
<tr>
<td>10</td>
<td>Modal Hammer</td>
<td>1200.8</td>
<td>5.67</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>30.8</td>
<td>0.018</td>
</tr>
<tr>
<td>100</td>
<td>3</td>
<td>37.7</td>
<td>0.037</td>
</tr>
<tr>
<td>100</td>
<td>4 (highest)</td>
<td>162.8</td>
<td>0.38</td>
</tr>
</tbody>
</table>

All of the filtered standard deviations in Table 14 are smaller than the initial standard deviations shown in Table 13. Again, for the automatic hammer, the standard deviation tends to increase as the force level is increased. Yet, even at the highest force level, the automatic hammer has much more reasonable maximum standard deviations of less than 0.4 N and the modal hammer has a standard deviation of approximately 5-10 N.
A modal model is deduced with a viscous damper for the first five elastic modes of vibration of the one beam mid-link structure as shown in Figure 40.
Figure 40: Modal energy dissipation and Frequency versus Modal force for the first five modes of the one beam test structure tightened to 5 in-lbs. The deduced analytical modal model with a viscous damper is plotted with a black dashed line, the measured data at three force levels is plotted with magenta (lowest force), red (middle force), and blue (highest force).

The comparison of the optimized analytical model show that the model is able to capture the response of the first five modes. Note that the optimization procedure fits the modal slip force term, $\hat{F}_s$, very close to the highest force level applied.
Table 15: Optimized parameters for a modal Iwan model with a viscous damper. First three elastic modes each at varying bolt torques.

<table>
<thead>
<tr>
<th>Bolt Torque (in-lbs)</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; Elastic Mode</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; Elastic Mode</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt; Elastic Mode</th>
<th>4&lt;sup&gt;th&lt;/sup&gt; Elastic Mode</th>
<th>5&lt;sup&gt;th&lt;/sup&gt; Elastic Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.60</td>
<td>2.97</td>
<td>0.58</td>
<td>11.7</td>
<td>0.45</td>
</tr>
<tr>
<td>10</td>
<td>1.68·10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>3.10·10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>2.06·10&lt;sup&gt;5&lt;/sup&gt;</td>
<td>1.48·10&lt;sup&gt;5&lt;/sup&gt;</td>
<td>1.39·10&lt;sup&gt;5&lt;/sup&gt;</td>
</tr>
<tr>
<td>6.17·10&lt;sup&gt;5&lt;/sup&gt;</td>
<td>6.07·10&lt;sup&gt;5&lt;/sup&gt;</td>
<td>4.46·10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>4.35·10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>1.79·10&lt;sup&gt;7&lt;/sup&gt;</td>
<td>1.80·10&lt;sup&gt;7&lt;/sup&gt;</td>
</tr>
<tr>
<td>-0.41</td>
<td>-0.825</td>
<td>-0.800</td>
<td>-0.997</td>
<td>-0.995</td>
<td>-0.959</td>
</tr>
<tr>
<td>1.06</td>
<td>2.35</td>
<td>5.85</td>
<td>2.42</td>
<td>1.82</td>
<td>5.75</td>
</tr>
</tbody>
</table>

The $\hat{\chi}_{\text{initial}}$ values show that the only mode with any considerable amount of nonlinear damping is the 1<sup>st</sup> mode. All of the higher frequency modes are may be accurately modeled with a linear viscous elastic modal model.
APPENDIX B: Two Beam Velocity FFTs

The fast Fourier transform (FFT) is found for the left end point of the beam for all the force levels at the three different torque levels.

![End Point FFT of Two Beam Structure at 10 in-lbs Torque](image_url)

**Figure 41:** FFT of the two beam structure at 10 in-lbs torque.

In Sec. 8.7, modal models were found for the first three elastic modes of the structure at frequencies of approximately 121, 214, and 475 Hz respectively. The FFTs from the end point show higher frequency modes being active; however, when a composite velocity frequency response function is plotted, the higher frequency modes have less response. From the various force levels applied, the higher frequency modes show a greater shift in frequency than the lower frequency modes that were studied. Notice the first mode seems to have been heavily damped from the 1 force measurement.
The FFTs at various force levels are also plotted for the structure when the bolts are tightened to 30 in-lbs. The addition of a modal hammer hit shows that as the input force increases, the damping become very large for this system. Comparing with Figure 41, the peaks seem to shift less as the force is increase since the bolts of this structure are tightened to a higher level.
The FFTs at various force levels are also plotted for the structure when the bolts are tightened to 50 in-lbs. Notice the damping of the 1st mode is much less than the other two bolt torques. This suggests that as the bolts are tightened, the damping of the system may be primarily due to sources of damping other than the bolts, i.e. material damping or damping from the boundary conditions.