Introduction to Iwan Models and Modal Testing for Structures with Joints

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Motivation

- Much of the damping in built up structures comes from the interfaces (joints).
- Joints are nonlinear and influenced by physics at multiple length scales.
- However, the response of structures with joints is often quasi-linear.

Thanks to Brandon Deaner (now at Mercury Marine) for his work in this area and for creating the initial version of these slides.
Outline

- Discrete Damping Model
- Modal Damping Model
- Apply the Modal Damping Model
  - Academic System
  - Finite Element Model
  - Beam Structure
  - Exhaust Structure
- Conclusions

We want to accurately model the nonlinear damping seen in joints.

- Structures with joints exhibit increased damping.

Damping depends on the amplitude of the force
The nonlinear damping of joints is classified into two regions.

- "micro-slip" – nonlinearity associated with the slip region at the outskirts of the contact patch.

- "macro-slip" – occurs when the stick region vanishes and there is large relative motion between the two parts.

A Jenkins element is often used to provide stick-slip behavior.

- Consists of:
  - Coulomb Friction Damper
  - Linear Elastic Spring

- Provides Stick-Slip behavior
  - Stuck state has stiffness and no energy dissipation
  - Slipped state has no stiffness and energy dissipation
One approach is to model a joint with several Jenkins elements over an area.

- **Advantage**: Predictive!
- **Disadvantage**: Many degrees-of-freedom, no guarantee that power law behavior is captured.

A parallel arrangement of Jenkins elements is known as an Iwan model and is used to model both regions of slip.

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A 4-Parameter Iwan model is used to capture both micro- and macro-slip.

- The Iwan Model consists of spring and frictional damper elements in parallel.

\[ \rho(\phi) = R \phi[H(\phi) - H(\phi - \phi_{max})] + S \delta(\phi - \phi_{max}) \]

Population Density

We can represent the 4-Parameter Iwan model graphically to describe how stiffness and dissipation change with force level.

Converted to physical parameters:

\[ \{R, \chi, \phi_{max}, S\} \rightarrow \{F_S, K_T, \chi, \beta\} \]

This is the key to the Iwan model: it captures a specific evolution of damping and stiffness with amplitude that has been observed in many experiments!!
Stiffness changes are typically small – damping effect is of primary interest.

\[ m\ddot{x} + c\dot{x} + kx = F_0 \cos(\omega t) \]

\[ \zeta = \frac{E_{\text{diss/cyc}}}{\left(2\pi\omega_a\omega_n|X|^2\right)} \]

\[ E_{\text{diss/cyc}} = \pi\omega_c |X|^2 \]

\[ \zeta = \left(\frac{1}{2}\left(V(t)^2 - V(t + \frac{2\pi}{\omega_n})^2\right)\right) \]

\[ \frac{2\pi\omega_a\omega_n |X|^2} \]

\[ \text{Slope} = 1 + \chi \]

\[ \text{Slope} = -1 \]

**Why would we want to apply the damping in a modal framework?**

- Simplify computations for structure with many joints

Many parameters needed to characterize each joint separately.

\[
\begin{align*}
F^1_S, K^1_T, \chi^1, \beta^1 \\
F^2_S, K^2_T, \chi^2, \beta^2 \\
F^3_S, K^3_T, \chi^3, \beta^3 \\
\vdots
\end{align*}
\]
Why would we want to apply the damping in a modal framework?

- Simplify computations for structure with many joints

\[ M = 1 \]

\[ \dot{q} + \alpha_{\infty} q = \Phi F^X + \tilde{F}_x^j \]

Because tests in the micro-slip regime frequently reveal that the structure as a whole often behaves modally!
The proposed modal Iwan model comes with several assumptions.

- Linear Modes of the system are preserved
  - No geometric nonlinearities due to large deflection or material nonlinearities.

- No coupling among modes
  
  \[ r^{th} \text{ Modal Force applied} = r^{th} \text{ Modal Response only} \]

Does the modal approach work for a simple mass spring system?

- Discrete Model:

- Modal Models:

Comparison procedure for discrete and modal simulations.

Discrete Simulation: $F_s, K_T, x, \beta$

Apply Impact Force

Convert physical response to modal

Hilbert Transform

Optimize Analytical Model to frequency and energy dissipation to each mode.

Min $f$

Modal Simulation: $F_s, K_T, x, \beta$

Apply Impact Force

Hilbert Transform

Hilbert Transform with polynomial fitting is used to extract frequency and energy dissipation data.

Instantaneous damping and frequency are related to the derivative of the amplitude and phase, respectively.
Optimization can be used to fit the analytical model to measured changes in dissipation and frequency.

\[
\text{Min } f = f_D + f_f = \left( \frac{\hat{D}_{\text{Exp}} - \hat{D}_{\text{Model}}}{\max(\hat{D}_{\text{Exp}} - \hat{D}_{\text{Model}})} \right)^2 + \left( \frac{\hat{f}_{\text{Exp}} - \hat{f}_{\text{Model}}}{\max(\hat{f}_{\text{Exp}} - \hat{f}_{\text{Model}})} \right)^2
\]

How well does a modal Iwan model predict the response of a spring mass system.
How well does a modal Iwan model predict the response of a spring mass system.

Limitations

- The previous results all excited the system with a force that was spatially distributed to excite a single linear mode.
- Forces with other spatial distribution can cause macro-slip at a lower force level.
Limitations – Mode 2

Frequency vs Log Modal Amplitude

Damping Ratio vs Log Modal Amp.

Limitations – Mode 3

Frequency vs Log Modal Amplitude

Damping Ratio vs Modal Amplitude
Can an Iwan model be used to predict the response of a FE model?

- **Academic simulations:**
  - micro-slip region = Excellent Representation
  - transition region = may be inaccurate
  - macro-slip region = Excellent Representation

- **Finite Element models?**

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**Comparison procedure for discrete and modal simulations.**

**Discrete Simulation:**

1. Apply Impact Force
2. Convert physical response to modal
   \[ q = \Phi^* \mathbf{x} \]
3. Hilbert Transform
4. Optimize Analytical Model to frequency and energy dissipation to each mode.
   \[ \min f \]
5. Compare Modal and Discrete simulations
How is a discrete Iwan joint applied in a finite element model?

- FE model
- Determine the contact surfaces
- Use an Iwan model for each joint

Mesh:
- Hexahedron elements
- Relatively coarse mesh (1,554 nodes)

Constraints:
- Free-Free Boundary conditions
- Two washers are merged with the link
- Washers are constrained to move and rotate along the surface of the beam

A simple FE model was created for simulations.
"For Simulations" isn't too specific. How to improve? "To evaluate the modal approach" ??

Did you sufficiently explain the motivation - Why did we create an FEA model? What are we hoping to learn from these simulations?

Matt Allen, 1/31/2013
How well does the modal Iwan model predict the response of the structure?

Mode 2

Finite Element Model 1st Mode
Finite Element Model 2nd Mode

Finite Element Model 1st Mode
Finite Element Model 2nd Mode
How well does the modal Iwan model predict the response of the structure?

**Mode 2**

Finite Element Model 2nd Mode

Modal Force

Modal Energy Dissipation/Cycle

**Mode 3**

Finite Element Model 3rd Mode

Modal Force

Modal Energy Dissipation/Cycle
The response can be simulated with a few SDOF modal simulations and compare well in the micro-slip region.

Modal Responses

Physical Responses
Can a modal Iwan model be used to predict the response of actual experimental data.

- FE simulations:
  - micro-slip region = Good Representation
  - macro-slip region = Fair Representation

- Laboratory data?

Laboratory tests were initially conducted on a beam with a link attached in the middle.

- Same dimensions as the FE model
- Washers between link and beam were removed

*The damping of this structure was found to be very small (joint carries little load) and hence a modified structure was studied instead.*
Alternative: Two-beam test structure

**Beam Suspension**
- 2 strings
- 8 elastic bungees

**Forcing**
- Automatic hammer
- Consistent force input

The two beam test structure shows measurable, nonlinear damping.

<table>
<thead>
<tr>
<th>Elastic Mode #</th>
<th>10 in-lbs Torque, $\zeta$ (%)</th>
<th>Percent Difference (%)</th>
<th>30 in-lbs Torque, $\zeta$ (%)</th>
<th>Percent Difference (%)</th>
<th>50 in-lbs Torque, $\zeta$ (%)</th>
<th>Percent Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>7400</td>
<td>0.29</td>
<td>1712</td>
<td>0.16</td>
<td>900</td>
</tr>
<tr>
<td>2</td>
<td>0.57</td>
<td>900</td>
<td>0.48</td>
<td>742</td>
<td>0.26</td>
<td>356</td>
</tr>
<tr>
<td>3</td>
<td>0.31</td>
<td>605</td>
<td>0.16</td>
<td>264</td>
<td>0.11</td>
<td>150</td>
</tr>
</tbody>
</table>
Measurements were taken with a scanning laser Doppler vibrometer.

**Velocity Measurements:**
- Polytec OFV-534
  - Single Point LDV used as reference.
- Polytec PSV-400
  - Scanning Head used to measure 70 points on the structure.

**Comparison procedure for discrete and modal simulations.**

**Laboratory Measurements:**
- Apply Impact Force
- Convert physical response to modal
- Hilbert Transform
- Optimize Analytical Model to frequency and energy dissipation to each mode.
- Min $f$

**Modal Simulation:**
- Apply Impact Force
- Hilbert Transform
- Compare Modal and Discrete simulations
Compare the standard deviation of the trimmed set versus untrimmed set.

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>20.2</td>
<td>0.80</td>
<td>0.088</td>
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<tr>
<td>10</td>
<td>2</td>
<td>32.8</td>
<td>0.27</td>
<td>0.025</td>
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<td>10</td>
<td>3</td>
<td>86.4</td>
<td>0.68</td>
<td>0.041</td>
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<td>10</td>
<td>4</td>
<td>288.6</td>
<td>6.10</td>
<td>0.213</td>
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<td>30</td>
<td>1</td>
<td>24.1</td>
<td>0.38</td>
<td>0.013</td>
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<td>30</td>
<td>2</td>
<td>30.9</td>
<td>0.51</td>
<td>0.019</td>
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<tr>
<td>30</td>
<td>3</td>
<td>52.8</td>
<td>3.84</td>
<td>0.125</td>
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<td>30</td>
<td>4</td>
<td>180.1</td>
<td>58.24</td>
<td>1.585</td>
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<tr>
<td>30</td>
<td>Modal Hammer</td>
<td>1444.5</td>
<td>139.34</td>
<td>3.081</td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>20.8</td>
<td>0.44</td>
<td>0.009</td>
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<tr>
<td>50</td>
<td>2</td>
<td>36.5</td>
<td>0.28</td>
<td>0.005</td>
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<td>50</td>
<td>3</td>
<td>60.3</td>
<td>0.61</td>
<td>0.011</td>
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<tr>
<td>50</td>
<td>4</td>
<td>238.6</td>
<td>15.30</td>
<td>0.310</td>
</tr>
<tr>
<td>50</td>
<td>Modal Hammer</td>
<td>1392.1</td>
<td>172.48</td>
<td>3.225</td>
</tr>
</tbody>
</table>

Next the measurements were band pass filtered. To isolate the modes, measurements were divided by the mass normalized mode shape value.
The modal Iwan model seems to have difficulty capturing the energy dissipation and frequency.

How do we account for the damping associated with the material and boundary conditions?

- Add a viscous damper to the modal Iwan model

![Diagram of modal Iwan model with viscous damper]
The modal Iwan model with a viscous damper captures the laboratory data accurately!

The response can then be simulated as a superposition of nonlinear SDOF systems.
These concepts also hold for realistic hardware

- Front and rear catalytic converters assembled together with required assembly torque and exhaust manifold gasket.
- System hung freely suspended by bungee cords to complete a roving hammer test.

Linear Experimental Modal Analysis

<table>
<thead>
<tr>
<th>Modal Index</th>
<th>Natural Frequency [Hz]</th>
<th>Damping Ratio</th>
<th>Deflection Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>113.70</td>
<td>0.0030</td>
<td>Bending</td>
</tr>
<tr>
<td>2</td>
<td>175.42</td>
<td>0.0043</td>
<td>Bending</td>
</tr>
<tr>
<td>3</td>
<td>243.41</td>
<td>0.0005</td>
<td>Localized Heat Shield Mode</td>
</tr>
<tr>
<td>4</td>
<td>247.38</td>
<td>0.0004</td>
<td>Localized Heat Shield Mode</td>
</tr>
<tr>
<td>5</td>
<td>262.71</td>
<td>0.0044</td>
<td>Torsion</td>
</tr>
<tr>
<td>6</td>
<td>348.68</td>
<td>0.0045</td>
<td>Bending</td>
</tr>
</tbody>
</table>

- Force levels kept low to estimate linear damping.
Modal Iwan model accurately captures the damping versus amplitude for various input points (various combinations of the different modal amplitudes)!

- Because the quasi-modal framework is valid, we can reduce the number of tests and test/model comparisons dramatically!

Preview: Modal Iwan parameters can(*) be used to deduce discrete joint parameters.

- Measurements of modal response in micro-slip regime used to deduce discrete joint properties. [Segalman, Allen, Eriten & Hoppman, ASME-IDETC 2015]
Conclusions

- Modal Iwan models can represent real structures quite accurately!
  - **Academic Model**
    - micro-slip region = Excellent Representation
    - macro-slip region = Excellent Representation
  - **FE Model**
    - micro-slip region = Good Representation
    - macro-slip region = Fair Representation
  - **Laboratory Data**
    - micro-slip region = Excellent Representation
    - macro-slip region = Not Obtained

Acknowledgements

Some of the work described was conducted at Sandia National Laboratories. Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. Special thanks to…

**Simulation Work:**
Michael Starr, Daniel Segalman, Michael Guthrie, Brandon Deaner, Robert Lacayo

**Lab Work:**
Jill Blecke, Matthew Allen, Hartono Sumali, Michael Starr, Randy Mayes, Brandon Zwink, Patrick Hunter, Dan Segalman, Steve Myatt

**Univ. Wisconsin Research Group:**
*Former:* Brandon Deaner, Robert Kuether, Hamid Ardeh, Dan Roettgen, David Ehrhardt, *Current:* Robert Lacayo, Kurt Hoppman, Melih Eriten, …


Alternative T/F Analysis: Zeroed Early-time FFT

- Nonlinearity is assumed to be active at high amplitudes and inactive at lower amplitudes.
- The response then becomes more linear as more of the initial nonlinear response is nullified.
- Impulse responses with initial segments of varying length set to zero are compared in the frequency domain.
- The nonzero portion of each impulse response begins at a point in which the response is near zero.

High Amplitude Nonlinear

Low Amplitude Linear
Beam: ZEFFTs

1400 and 2750 Hz modes clearly soften as the amplitude increases.

FEA Mode 1: 644 Hz

FEA Mode 3: 1581 Hz

FEA Mode 6: 3035 Hz

All of these lines come from one response with different parts erased!

Beam: BEND and IBEND

IBEND:
- Somewhat nonlinear before 70 ms,
- Significantly nonlinear before 50 ms.
“Linear Beam”

- Results on previous slides for 200 lbf peak force
- This shows the ZEFFTs for a 40 lbf peak force
- 3 Hz shift observed from high to low amplitude.