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EXPERIMENTAL ASSESSMENT OF JOINT-LIKE MODAL MODELS FOR STRUCTURES

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ABSTRACT

Several of the authors, and others, have explored the use of modal-like models for structures having nonlinearities associated with compressive joints (such as bolted connections.) In these models, the deformations are treated as consisting of modal expansions, but with the modal coordinates that evolving nonlinearly over time. There have been several theoretical treatments of this strategy and the authors report here on an early effort to confirm the approach through experiment.

For this purpose simple structures consisting of pairs of plates were assembled using four bolted connections. The structures were excited by modal hammer at various locations and the modes identified by scanning laser vibrometer. In each case, multiple modes were excited and the evolution of modal coordinates was achieved by band-pass filtering at the relevant frequencies.

Making some simple kinematic assumptions about relative deformations of the component plates and exploiting symmetries permits the mapping from ring-down of each mode to constitutive behavior of each joint. If the strategy of using joint-like modal models for bolted structures is valid, the joint constitutive models deduced from any mode should be adequate to predict the apparent, but nonlinear modal behavior at other resonances. Multiple test specimens were manufactured to assess this predictive capability in the context of part-to-part variability intrinsic to the dynamics of such structures.

INTRODUCTION

One of the perpetually intriguing and frustrating aspects of structural dynamics if how damping is to be integrated into the models. There is extensive and classical literature about how to incorporate linear damping into M,C,K mass-damping-stiffness formalisms. These include mass-proportional and stiffness-proportional damping, and modal damping. Additionally, more complex formalisms for the damping matrix can be derived to accommodate effects of general linear viscoelasticity (1) or even rotational dynamics (2).

The issue becomes much more complex when we acknowledge the nonlinearity intrinsic to real structures. (By structures, we mean conventional structures assembled from multiple components by nuts and bolts, screws, rivets, etc.) There is substantial literature on the nonlinearity observed in structures and we site just a few (3-6). Among the qualitative nonlinearities observed on mechanical joints and jointed structure are:

1. Nonlinear damping. For a linear system the dissipation should grow quadratically with imposed force, but much faster growth of dissipation with applied load is found experimentally.
2. Softening with applied loads. This is illustrated most clearly with harmonic excitation where the resonant frequencies decrease with force amplitude.
3. Macro-slip at high loads.

The first two of these features is explored systematically using a resonant device such as shown in the following figure.

Fig. 1: Photograph of experimental setup used to measure dissipation in a lap joint. A finite element representation of the joint itself is shown on the right.

This particular facility was devised by Gregory (4) and exploits the same physics as one developed by Gaul and colleagues earlier (6). The nonlinear damping is illustrated in the following figure from (4) comparing the dissipation in jointed and unjointed specimens at different load amplitudes. The mechanics of this test specimen is detailed in (4).

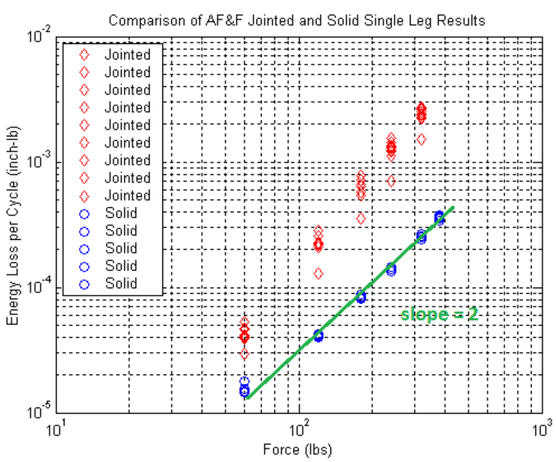

Fig. 2: Energy dissipation vs. Force measured from the joint shown in Fig. 1.

As expected, the solid specimens manifest much less dissipation than do the jointed specimens and the dissipation associated with the solid specimens comply with a power-law relationship with exponent 2. The jointed specimens conform to a power law slope greater than 2. In general individual joints manifest a power-law slope between 2.3 and 2.9. (4)

The softening and macro-slip properties of bolted joints are also broadly discussed in the literature and are summarized well in (4). It is important in what follows that in the broad regimes where structural dynamics is applied, where vibration is significant but loads stay substantially below those necessary to cause macro-slip, it is the nonlinearity of damping that is most dramatic. These are also load regimes below those necessary for geometric nonlinearity besides those at the joint to manifest.

In the following, we explore some of the modeling approaches and assumptions that go into current strategies to incorporate observed nonlinearity into the structural dynamics models of built-up systems. Additionally, we explore some data from simple experiments to assess the validity of one of the most useful simplifying assumptions.

MODELS FOR JOINTS AND JOINTED STRUCTURES

The first step to modeling jointed structures would appear to be that of learning to model individual joints and then either incorporating those joint models directly into the structural model or using insights gained in this first step to suggest new strategies appropriate at the structural level. Let's examine individual joint models first.

Joint Constitutive Models

Joint specimens are themselves structures, so modeling of those specimens and comparison of the predictions of their models with experiment or deduction of model parameters from experiment requires some sort of separation between the interface phenomena and the rest of the specimen. For this the concept of a "whole joint" model is introduced. The approach described here is employed in FE modeling of large structures with small contact patches. The technique is to define a rigid surface (geometric patch) on each side of the interface and to slave each rigid surface to a single representative node. This concept is suggested in the following figure. The joint constitutive model then couples the forces (and moments) and displacements (and rotations) of those two representative nodes. In commercial FE code, one usually defines the rigid surfaces using rigid elements (such as RBE3) or multi-point constraints (MPCs).

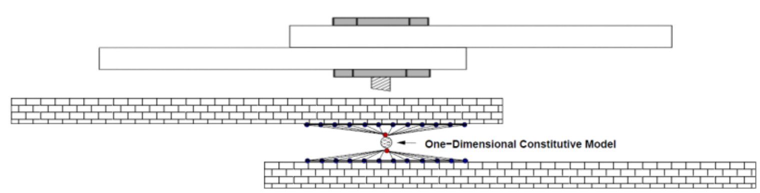

Fig. 3: Schematic showing how interfaces are modeled using a whole joint model.

In this approach the gross elasticity of the specimen is represented by the finite element mesh and the nonlinearity of the joint is captured by the scalar constitutive model connecting the opposing nodes.

Interestingly, the nonlinear behavior of mechanical joints is very similar to that of one dimensional metal elasto-plasticity. One of the simpler by still very general metal plasticity model is that associated with the names of Masing, Bausinger, Prandtl, Ishlinskii, and Iwan. Something of the history of this mathematical model is discussed in (7). The mathematical form is

$$F(t) = \int_0^\infty \rho(\phi)\left[u(t) - x(t,\phi)\right] d\phi \qquad (1)$$

Where F is the interface force, u is interface displacement, ρ is a kernel that characterizes the joint and x is a continuum of state variables that evolve as

$$\dot{x}(t,\phi) = \begin{cases} \dot{u} \ \text{ if } \ \left\| u - x(t,\phi) \right\| \\ \qquad \text{and } \ \dot{u}\left(u - x(t,\phi)\right) > 0 \\ 0 \quad \text{otherwise} \end{cases} \qquad (2)$$

This mathematical constitutive equation captures the behavior of a parallel arrangement of Jenkins elements as suggested by the following figure.

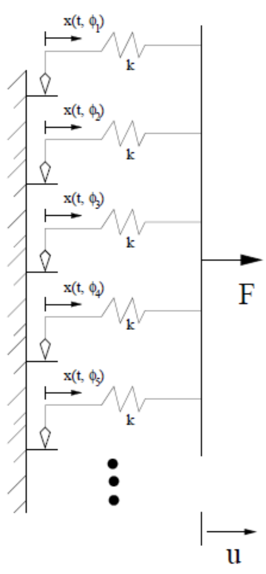

Fig. 4: Schematic of parallel arrangement of Jenkins elements.

Several choices of ρ can be chosen to reproduce the power-law dissipation observed experimentally and indeed the model parameters can be deduced in systematic manners from experimental data (8). This constitutive equation along with the whole-joint kinematics have been introduced into finite element structural dynamics code and have been used quite successfully to predict the response of test specimens.

Implementation of this approach to systems with many interfaces is probably impractical. There would far too many interfaces to parameterize the corresponding ρ through experiment. This is an argument that it is necessary to develop some other approach to capture the nonlinear behavior of complex structures.

The Structural Dynamics Paradox and a Hypothesis

This is a good stage to discuss an inconsistency between what we know about nonlinearity of real structures and how we, as a community, perform structural dynamics. In general structural dynamics is performed employing thoroughly linear models. In fact there are some very sophistocated methods for extrapolating damping matrices from structural experiments.

Surprisingly, these linear modeling endeavors are very successful. How can we reconcile the success of linear models with our observations of dramatic nonlinearity in real structures? The authors offer a hypothesis that might resolve this apparent inconsistency:

> *Real structures do behave nonlinearly, but are reasonably approximated in the regime of deformation amplitude by a linear model whose parameters are determined by that deformation amplitude.*

One ramification of this hypothesis is that linear models can be successful only when they are parameterized using data collected in the amplitude range at which predictions are required. We realize that this generally is the practice.

We also must assess what other hypotheses are necessary for the above to be consistent. One issues is that each experiment includes loadings containing many modal components, so we must assume that the nonlinear response is such that each modal response can be calibrated separtely. In other words, the nonlinear response of each mode is independent of the force/deformation amplitudes of the other modes. This suggests that we look for nonlinear modal models, meaning models where deformation is expressed as linear combinations of the modes of some underlying linear systems, but where the modal coordinates evolve in a nonlinear manner.

What kind of nonlinear modal model should we explore. One hint is that the underlying nonlinearity is that due to joints and the joint forces evolve as indicated in Equations (1) and (2). Another hint is that obtained by subjecting structures with a plethora of interfaces to harmonic excitation. In such cases we again see a power-law relationship between force amplitude and dissipation with exponents substantially larger than 2.

Modeling the Joint-Like Behavior of Structures

Here we develop a system of equations for the full structure representing the joint nonlinearity as an unknown force F_B^J :

$$M_{\mathrm{B}}\ddot{u}_{\mathrm{B}} + C_{\mathrm{B}}\dot{u}_{\mathrm{B}} + K_{\mathrm{B}}^{\infty}u_{\mathrm{B}} = F_{\mathrm{B}}^{X} + F_{\mathrm{B}}^{J} \tag{3}$$

where $M_B, C_B, \text{and}\, K_B^{\infty}$ are mass, damping, and stiffness matrices respectively and F_B^X is the vector of imposed loads. This matrix is diagonalized by the modal matrix Φ_B

$$\begin{aligned}&\ddot{q}_{\mathrm{B}} + \mathrm{diag}\left(\{2\zeta_k\omega_k\}\right)\dot{q}_{\mathrm{B}} + \mathrm{diag}\left(\{\omega_k^2\}\right)q_{\mathrm{B}} \\ &\quad = \Phi_{\mathrm{B}}^T F_{\mathrm{B}}^X + \Phi_{\mathrm{B}}^T \Delta F_{\mathrm{B}}^J\end{aligned} \tag{4}$$

where $\{q_B\}$ are modal coordinates. The assumptions of the previous section are now invoked. We assume that the nonlinear modal force associated with mode evolves only according to the deformation history of just that mode.

$$\begin{aligned}\left(\Phi_{\mathrm{B}}^T \Delta F_{\mathrm{B}}^J\right)_k &= f_k\left(q_k(s), s = -\infty\ \mathrm{to}\ t\right) \\ &= \int_0^{\infty} \rho_k(\phi)[q_k(t) - \gamma(t,\phi)]\, d\phi\end{aligned} \tag{5}$$

This leaves us with parametrizing a ρ_k for each mode, but this is done via modal testing on the structure and can be accomplished independently of the number of interfaces fundamentally responsible for the system nonlinearity. This form of constitutive model was explored in (9) and (10). For the moments, we refer to structural dynamics models of this sort as "modal Masing models".

TESTING THE MODAL MASING ASSUMPTION

The above sort of constitutive model for the overall structure is very appealing, but is it consistent with what can be observed experimentally. Let us consider the very simple structure consisting of two plates bolted together at three locations. This is sketched in the following figure:

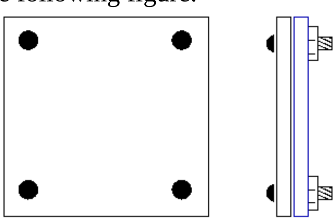

Fig. 5: Schematic of the plate assembly studied.

This simple structure has sufficient symmetries that we can learn a lot from some simple impact testing. The test specimen is supported on bungee (see below) and struck with an impact hammer. A Polytec PSV400 scanning laser vibrometer was used to identify modes and an Endevco 25B accelerometer was used to capture the ring-down of vibration.

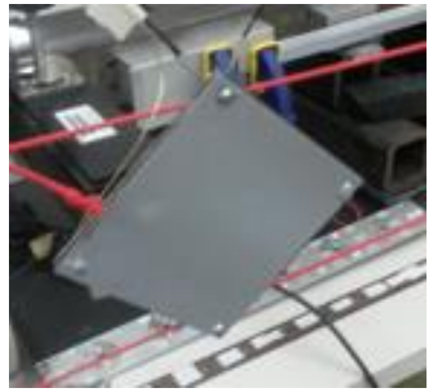

Fig. 6: Photograph of the plate specimen suspended by elastic cords for testing.

Using the Zeroed Early-time Fourier decomposition technique developed in (11), one obtains plots such as the following, showing how at early times damping is higher and the resonance frequency is lower than it is later on when the amplitude has diminished. These information-rich plots are obtained by performing Fourier analysis on data from different length time windows; analyses corresponding to time windows from t=0 out to short times are shown in blue and analyses corresponding to time windows from t=0 out to long times (indicated in the legend) are shown in red.

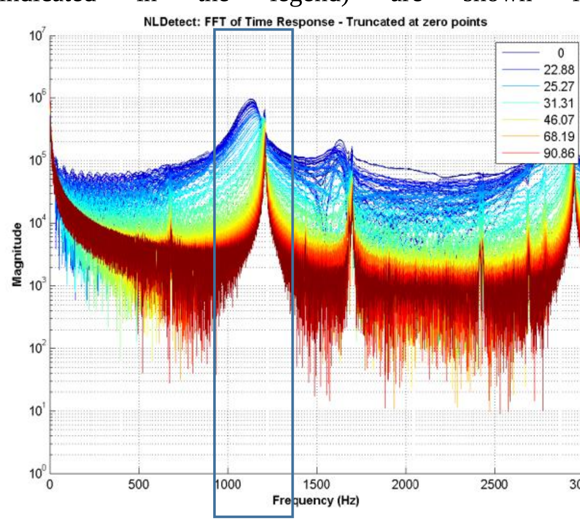

Fig. 7: Zeroed Early-time FFT of a representative measurement showing typical joint type nonlinearity. The Fourier analysis corresponding to a single deformation mode is indicated in the blue box.

This is very joint-like behavior and supportive or our hypothesis.

We combine these experimental results with some simple analytic modeling. First, we must make a very strong kinematic assumption. We assume that for all modes of interest to us, the plates deform in a mutually "spooning" manner, as shown below.

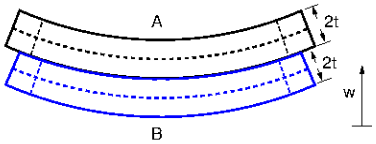

Fig. 8: Schematic showing how the plate kinematics were treated.

Finite element analysis is also helpful to us. Though finite element modeling of built-up structures is problematic, there is great confidence in modal analysis of individual monolithic plates.

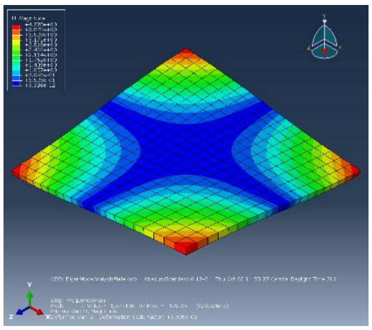

Fig. 9: Finite element model of the plate specimen.

With the spooning assumption we deduce that the composite plate will have the same modes as would each component plate vibrating on its own. (Of course the frequencies of the individual plates and the composite will be different.) Additionally with the spooning assumption and the finite element mode shapes we can deduce the discontinuity in displacement at the joints with plate displacement:

$$\vec{\Delta}_j = 2t\left(w_x\vec{i} + w_y\vec{j}\right) \tag{6}$$

where w_x and w_y are the rotations of the plate in the x and y directions and t is half the thickness as shown above. The dissipation, D_J, at each joint is presumed to be a power-law function of the shear force, F_J, there with exponent α.

$$D_J = \nu F_J^{\alpha} \tag{7}$$

Considering that we are dealing with small oscillations, the joint force and joint displacement are proportional

$$F_J = k\,\Delta_J \tag{8}$$

Given the resonance and ring down of any mode where all joints deform in the same amplitude, we can deduce all necessary parameters: k, ν, and α .

If our formulation and all our assumptions about modal independence are correct, we should be able to use one mode to deduce joint parameters and then predict the resonance frequencies and power-law behavior of any other mode. (For instance, the resonant frequency of a mode of the jointed structure can be expresses in terms of the resonance of an individual plate and the stiffness of the joints:

$$(\omega_k^B)^2 = \omega_k^2 + \frac{1}{2}\,k\sum_j \Delta_{j,k}^2$$

where ω_k^B is the kth natural frequency of the bolted assembly, ω_k is the corresponding stiffness of the individual plate, and $\Delta_{j,k}$ is the relative displacement of joint j corresponding unit displacement of the kth mode.)

We proceed by determining which modes to examine. The finite element-derived modes are show below. There are reasons why some of them cannot be used.

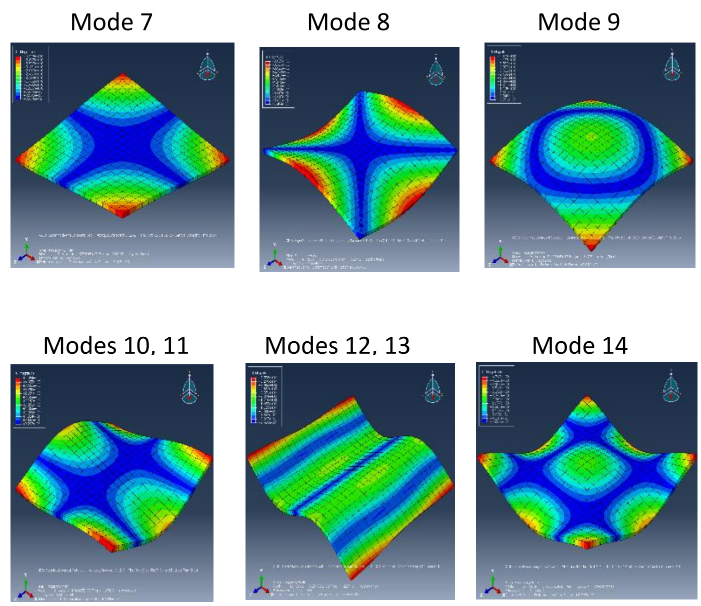

Fig. 10: FEA mode shapes of a single plate.

There is only one accelerometer and it lies along a nodal line of Mode 8; there is no data to be had for that mode. (Other locations for the accelerometer were considered, but the ultimate decision was made to maximize signal strength of the other modes.) Modes 10 and 11 occur at the same frequency and it is impossible to distinguish the mode displacement of either one of them using only one accelerometer. There is a similar problem with modes 12 and 13, so they can't be addressed in this particular study. That leaves Modes 7, 9, and 14. The measurements revealed that Mode 7 damps out so quickly that there were insufficient cycles to fit a power-law envelope. Hence, we use Mode 9 to calibrate our joints and use the resulting nonlinear modal model to predict the power-law dissipation of mode 14.

The plate was excited with a modal impact hammer with a rubber tip at various force levels ranging from approximately 300 to 600 N peak and the power-law plot of dissipation versus modal amplitude for Mode 9, the first breathing mode, was found and is shown in the following figure. The first number in each legend entry is the approximate force level in Newtons. We see that there is some sensitivity as to where the hammer hit occurs.

The notation (CC) and (TT) indicate that the excitation was at the center or near the top corner of the plate, respectively. The amplitude of the hit is indicated by its value in Newtons, such as 400N, 600N, etc. Perhaps the center hits also excite modes where the plates move opposite each other as opposed to spooning. From kinematic considerations, we would indeed expect that opposing motion would cause less relative displacement at the joints and hence less dissipation. It also may be possible that the initial shock to the plate induces a short macro-slip event that changes the ensuing energy dissipation.

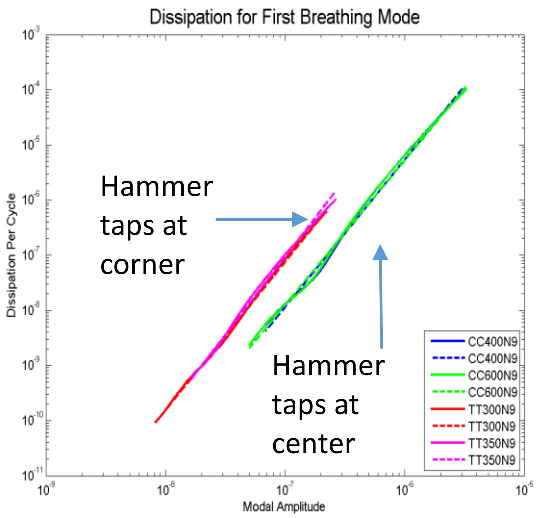

Fig. 11: Measured energy dissipation versus amplitude for Mode 9 for several trials at two locations.

We may now predict the dissipation Mode 14 (second breathing mode) and compare to the experimental values for that mode. Once again the modal energy dissipation was measured using excitations at the center (CC) and near the top (TT). The results are shown in the following figure. In this case there was less variation with excitation point.

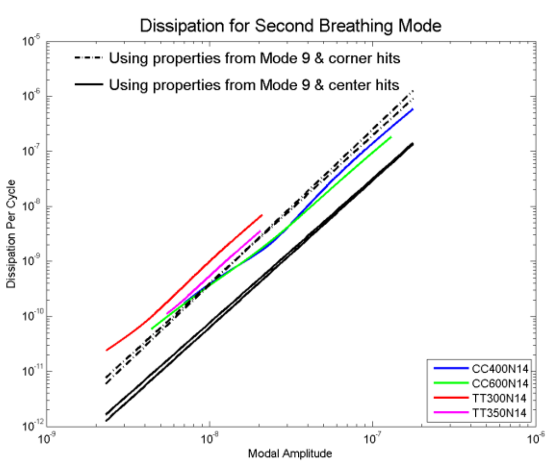

Fig. 12: Energy dissipation versus amplitude predicted for Mode 14.

We have two sets of predictions: those based on center hits and response of Mode 9 and those based on corner hits and response of Mode 9. We discover that predictions based on corner hits are actually very good approximations for the dissipation measured on Mode 14. We also discover that, while predictions employing data taken for Mode 9 and center hits seem to capture the power law slope accurately, there is an offset in the dissipation in those predictions (i.e. due to ν) that is not accurate. The authors suspect that the large systematic variation in the dissipation in Mode 9 is due to the strong excitation of two modes by center hits: one consistent with the kinematics assumed in Figure 8 and one inconsistent with those kinematics. The authors intend to address this possibility in planned experiments involving multiple accelerometers.

CONCLUSIONS

Though there some uncertainty associated with the kinematics of the problem, there is sufficient agreement between the predications derived using the key assumptions underlying modal Masing models and experimental ring-down data.

One potential source of error is the fact that the samples used to date were not machined with high precision, so the four joints may not be loaded equally as the plate deforms. New samples have been created and more extensive testing is planned on these to further explore these techniques. Of course more experiments on more interesting structures will be necessary before one can begin to have confidence in this approach. Additionally, some clarification on plate kinematics should be obtained by placing accelerometers on each plate and comparing phase.

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REFERENCES

1. Segalman, D. J. "Modelling joint friction in structural dynamics." *Structural Control and Health Monitoring*, 2006: 430-453.
2. Meirovitch, L. *Computational Methods in Structural Dynamics.* New York: Springer, 1980.
3. Goodman, L. E. "Contributions of continuum mechanics to the analysis of the sliding of." *AMD (Symposium Series of the ASME Applied Mechanics Division)* 39 (1980): 1 – 12.
4. D. J. Segalman, D. L. Gregory, M. J. Starr, B. R. Resor, M. D. Jew, J. P. Lauffer, and N. M. Ames. *Handbook on Dynamics of Jointed Structures.* Sandia Report, Albuquerque, NM: Sandia National Laboratories, 2009.

5. Nitsche, L. Gaul and R. "The role of friction in mechanical joints." *Applied Mechanics* 52 (2001): 93–106.
6. Lenz, L. Gaul and J. "Nonlinear dynamics of structures assembled by bolted joints." *Acta* 125 (1997): 169 – 181.
7. Daniel J. Segalman and Michael J. Starr, "Iwan models and their provenance." *ASME 2012 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference.* Chicago: American Society of Mechanical Engineers, 2012.
8. Segalman, Daniel J. "A four-parameter Iwan model for lap-type joints." *Journal of Applied Mechanics* (American Society of Mechanical Engineers), 2005: 752-760.
9. Segalman, D. J. *A Modal Approach to Modeling Spatially Distributed Vibration.* Sandia Report, Albuquerque, NM: Sandia National Laboratories, 2010.
10. Brandon J. Deaner, Matthew S. Allen, Michael J. Starr and Daniel J. Segalman. "Application of Viscous and Iwan Modal Damping Models to Experimental Measurements From Bolted Structures." *ASME Proceedings 25th Conference on Mechanical Vibration and Noise.* Portland: American Society of Mechanical Engineers, 2013. V008T13A020.
11. Mayes, Matthew S. Allen and Randall L. "Estimating the degree of nonlinearity in transient responses with zeroed early-time fast Fourier transforms." *Mechanical Systems and Signal Processing*, 2010: 2049-2064.

